

# REAL OPTIONS TO INVEST

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## 1. THE SET-UP

In the options-to-invest model discussed in these notes, I assume that the holder of the options permanently faces the decision on either to exercise the options (i.e., to execute the project and collect the resulting cash flows), or to continue to wait, thereby sacrificing the resulting cash flows, but keep the options to avoid unexpectedly low cash flows. I assume that the initial outlay ( $Y_0$ ) is deterministic and constant and that all uncertainty can be summarized by a simple geometric Brownian motion (GBM):

$$\frac{dP(t)}{P(t)} = \mu dt + \sigma dW(t) \quad (1)$$

where  $W(t)$ , for  $t > 0$ , is a real Brownian motion, and  $P(t)$  is a stochastic output price variable.

In keeping with much of the literature, I assume that exercising the option to invest is irreversible; that is, I assume that it is not possible to abandon a project before its terminal date ( $T$ ), where, for expositional clarity, we take  $T$  to be infinite throughout the notes and consider a perpetual option to invest. In addition, I also assume, for simplicity, that the project's only benefit is materialized through its expected cash flow, where the cash flows,  $CFO$ , is a function of the stochastic variable,  $P(t)$ , and time,  $t$ ; that is,  $CFO(t) = CFO(P(t), t)$ .

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This paper was written as a result of the many discussions I had with my colleague and co-author: Margaret Insley, during the time when we wrote our paper on: *Contrasting Two Approaches in Real Options Valuation: Contingent Claims versus Dynamic Programming* (2008). I would like to thank her for the discussions, without implicating her for any remaining errors.