3.2 Non-renewable Resources

A. Are stocks of non-renewable resources fixed?

Reserves measures have an economic component – eg. what could be extracted at current prices?

- Location and quantities of reserves of resources like oil and gas are not known – and can be expanded with increased exploration effort.

- Base resource: the mass that is thought to exist in the earth's crust

- Resource potential: That which could be extracted given current and expected technologies

- World reserve base: estimates of the upper bounds of resource stocks (including reserves not yet discovered) that are economically recoverable under reasonable expectations of future prices, costs, and technological possibilities

- Reserves: quantities that are economically recoverable given present costs and prices

See Perman, p 509.
Also the BP Statistical Review of World Energy, 2004, has useful information on reserves versus production for various non-renewable energy sources.

B. A two period model

Two periods: period 0 and period 1

A fixed stock of known size of a non-renewable resource

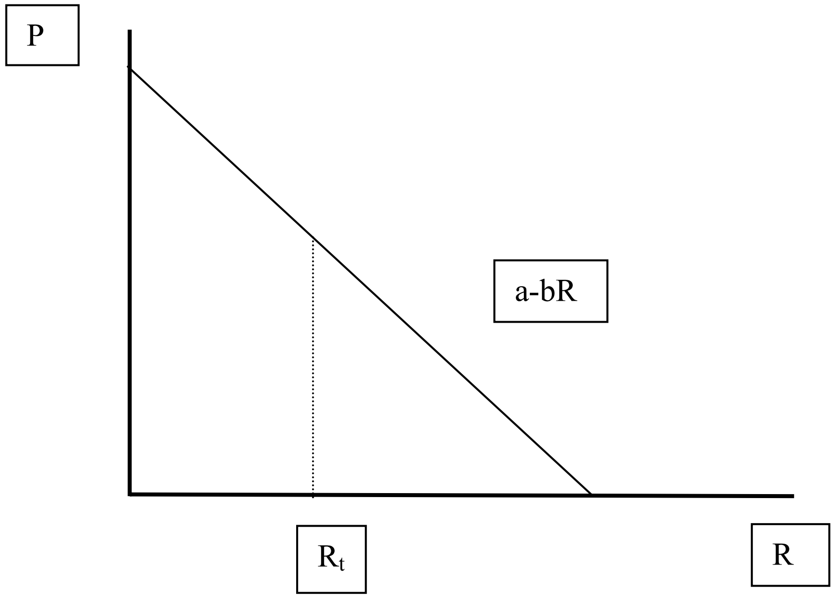
Initial stock at period 0 is $\overline{S}$.

R(t) is extracted in period t.

Inverse demand function: $P_t = a - bR_t$

Demand goes to zero at price *a*.

Resource is either non-essential or it possesses
a substitute.



Total benefit to consumers from consuming R_t:

$$B(R_t) = \int_0^{R_t} (a - bR)dR$$
$$= aR_t - \frac{b}{2}R_t^2 \quad (2.1)$$

Extractions costs:

$$C_t = cR_t \quad (2.2)$$

Total net social benefit:

$$NSB_t = B_t - C_t \quad (2.3)$$

$$NSB(R_t) = aR_t - \tfrac{b}{2}R_t^2 - cR_t \quad (2.4)$$

Assume the social utility in each period is equal to the net social benefits in each period (a strong assumption).

Optimization problem:

$$\underset{R_0, R_1}{Max} W = NSB_0 + \frac{NSB_1}{1+\rho} \tag{2.5}$$

subject to:

$$R_0 + R_1 = \overline{S} \tag{2.6}$$

Set up the Lagrangian and optimize with respect to R_0 and R_1.

Try this yourselves.

We get a two period version of Hotelling's rule:

$$\rho = \frac{(P_1 - c) - (P_0 - c)}{(P_0 - c)} \tag{2.7}$$

An efficient extraction profile requires the net price of the resource to grow at the social discount rate.

The optimal extraction program also requires that the two gross prices, P_0 and P_1, satisfy the following:

$$\begin{aligned} P_0 &= a - bR_0 \\ P_1 &= a - bR_1 \\ R_0 + R_1 &= \overline{S} \\ (P_0 - c)(1 - \rho) &= (P_1 - c) \end{aligned} \tag{2.8}$$

These conditions will uniquely define the two prices and quantities of resources to be extracted for welfare maximization.

Try problem 1 in Perman, p. 533.

Note on transversality conditions with a non-negativity constraint on the stock:

T free; $x(T) \geq x_{\min}$

(i) $x^*(T) > x_{\min}$ then x(T) is free and $\lambda(T)=0$ or $\lambda(T)=F'(x(T))$ (F is scrap value).

OR

(ii) $x^*(T)=x_{\min}$, then x(T) is fixed and $\lambda(T)$ is free.

In summary:

$$\lambda(T) \geq 0,$$

$$x(T) \geq x_{\min},$$

$$\lambda(T)[x(T) - x_{\min}] = 0$$

Also note that for infinite horizon problems for the transversality conditions, we take the limit as $T \to \infty$.

Assume:
- a finite stock of the resource
- a social planner
- it costs nothing to extract the resource
- demand functions:

$$R(t) = D(P(t))$$
$$P(t) = D^{-1}(R(t)) \quad \text{inverse demand function} \tag{2.10}$$

Flow of social welfare at a point in time:

$$W(t) = \int_{s=0}^{s=R(t)} P(s)\,ds \tag{2.11}$$

Objective:

$$\max J = \int_0^T e^{-\rho t} W(t) dt$$
$$= \int_0^T e^{-\rho t} \left\{ \int_0^{R(t)} p(s) ds \right\} dt \tag{2.12}$$

Subject to:

$$\dot{S} = -R(t)$$
$$S(0) = S_0 \tag{2.13}$$
$$S(t) \geq 0,\ R(t) \geq 0$$

Find the current valued Hamiltonian. We use λ to denote the current valued costate variable in this section.

$$\tilde{H} = \int_0^{R(t)} P(s) ds + \lambda(-R(t)) \tag{2.14}$$

FOC's:

$$\frac{\partial \tilde{H}}{\partial R} = P(R(t)) - \lambda(t) = 0 \qquad (2.15)$$

$$\dot{\lambda} - \lambda\rho = -\frac{\partial \tilde{H}}{\partial S} = 0$$
$$\Rightarrow \dot{\lambda} = \lambda\rho \qquad (2.16)$$

Solving Equation (2.16):

$$\lambda(t) = \lambda(0)e^{\rho t} \qquad (2.17)$$

From Equation (2.15),

$$P(t) = P(0)e^{\rho t} \qquad (2.18)$$

$$\dot{P}(t) = \rho P(t) \qquad (2.19)$$

This is Hotelling's rule again.

Transversality conditions:

$$\lambda(T)S(T) = 0 \qquad (2.20)$$
$$H(T) = 0 \qquad (2.21)$$

We can conclude that S*(T)=0. It is optimal to exhaust the resource.

Proof by contradiction:

Suppose S*(T)>0. Then holding T constant, we could have increased consumption R*(T), and this would make social welfare higher since there are no cocts to consumption. Thus it cannot be optimal to leave S*(T)>0. Hence S*(T)=0.

To find the optimal T, we want the Hamiltonian at T to be exactly zero:

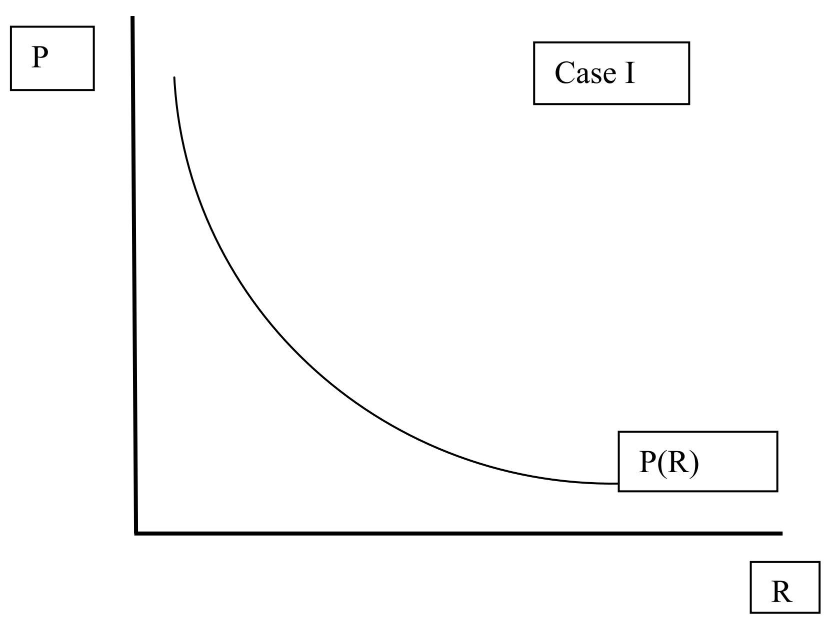
$$H(T) = 0$$

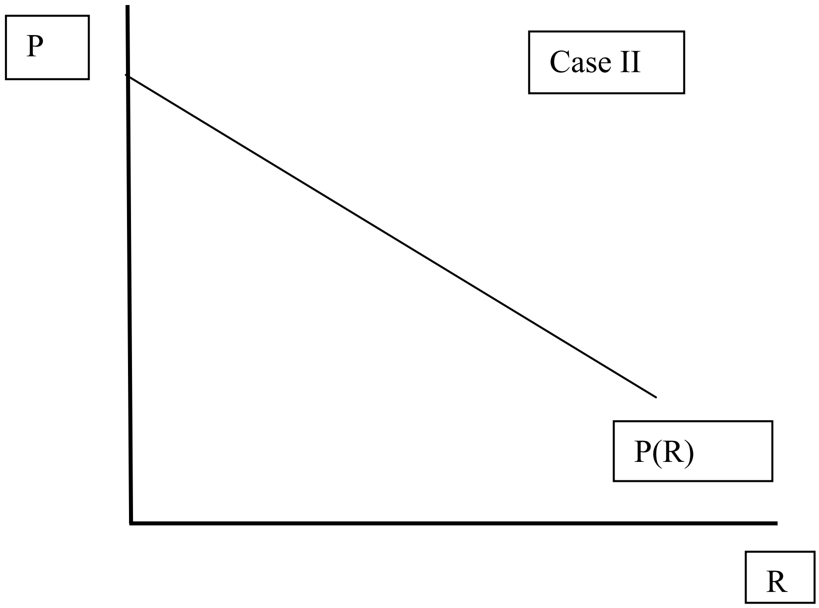
$$\int_0^{R(T)} P(s)ds - \lambda(T)R(T) = 0 \qquad (2.22)$$

Using the FOC's we can write:

$$\int_0^{R(T)} P(s)ds = P(T)R(T) \qquad (2.23)$$

What happens to P as the resource is exhausted? Two possibilities:





Case I: $\lim_{R \to 0} P(R) = \infty$; R*(T)=0. If we are to finish with a zero rate of extraction, price must rise infinitely high. This will take an infinite amount of time. Our optimal time horizon is T→∞.

Case II: $\lim_{R \to 0} P(R) = K$ where K is a choke price. R(T)=0, P*(T)=K. There will be a finite terminal time. A backstop technology creates a ceiling on the price of the non-renewable resource.

Case I:

$$\begin{aligned} &T^* \to \infty \\ &P(T) = P(0)e^{\rho T} \\ &P(t) = P(0)e^{\rho t} \\ &S^*(T) = 0 \end{aligned} \tag{2.24}$$

$$\int_0^{\infty} R(t)dt = S_0 \tag{2.25}$$

Equation (2.25) follows because total consumption must equal the stock of the resource.

This allows us to solve for a particular P(0)*. How?
We know R(t)=D(P(T))=D(P(0)$e^{\rho t}$). Substitute into Equation (2.25) gives:

$$\int_0^{\infty} D(P(0)e^{\rho t})dt = S_0 \tag{2.26}$$

This is one equation in one unknown. It may be difficult to solve – could be highly non-linear.

Case II:

$$\begin{aligned} P(T) &= K \\ P(t) &= P(0)e^{\rho t} \\ S^*(T) &= 0 \end{aligned} \tag{2.27}$$

We don't know T or p(0). We can use the first two equations of Equation (2.27) to solve for P(0).

$$P(0) = Ke^{-\rho T} \tag{2.28}$$

Then we solve for T* using:

$$\int_0^{T} R(t)dt = S_0 \tag{2.29}$$

$$\begin{aligned} R(t) &= D(P(t)) \\ P(t) &= Ke^{-\rho(T-t)} \\ R(t) &= D(Ke^{-\rho(T-t)}) \end{aligned} \tag{2.30}$$

We can substitute for R(t) in Equation (2.29) and solve for T*. This gives us the time to switch to the backstop technology.

Example

Suppose the inverse demand curve is:

$$P = Ke^{-aR} \tag{2.31}$$

Note that K is the choke price.

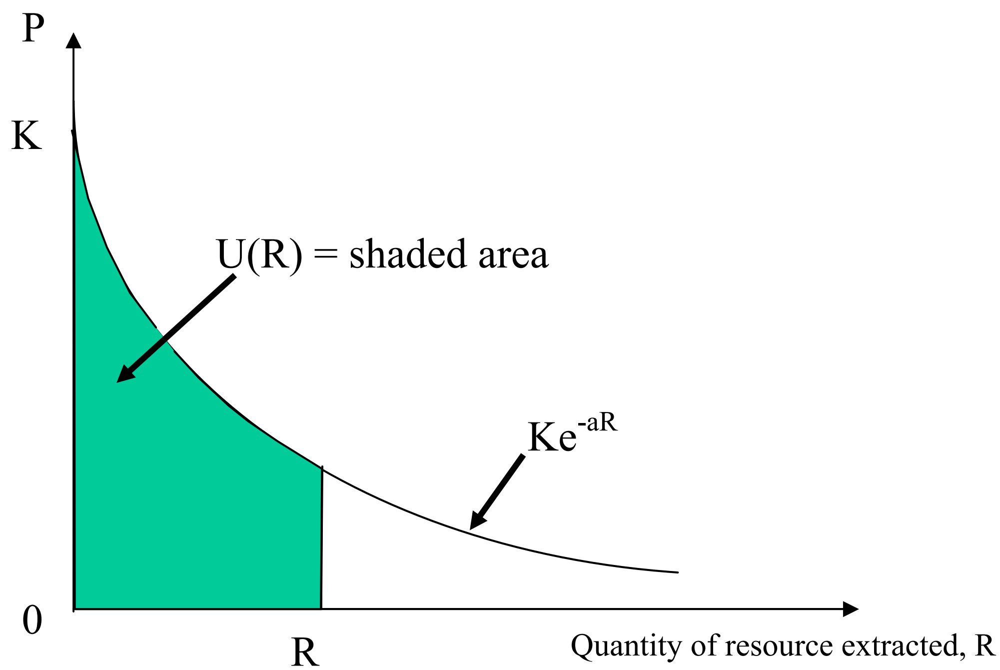
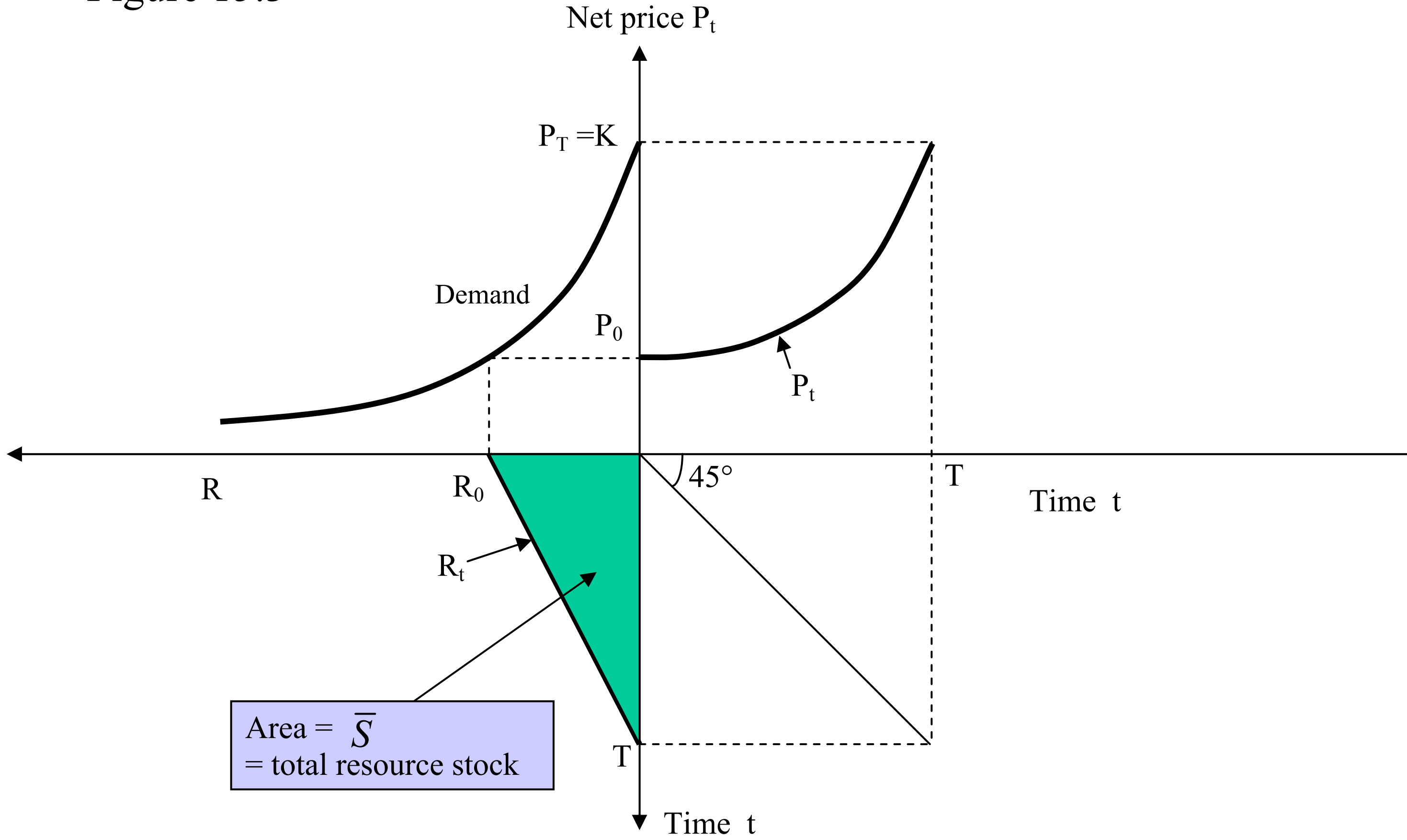


Figure 15.2 A resource demand curve, and the total utility from consuming a particular quantity of the resource.

Solve for P*(0), R*(t), T*, R*(0).

Figure 15.3


Net price P_t
P_T =K
Demand
P_0
P_t
R
R_0
45°
T
Time t
R_t
Area = $\overline{S}$
= total resource stock
T
Time t