An Options Pricing Approach to Ramping Rate Restrictions at Hydro Power Plants

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Abstract

This paper uses a real options approach to examine the impact of ramping rate restrictions imposed on hydro operations to protect aquatic ecosystems. We consider the effect on hydro plant value in order to inform policy decisions. A novelty of the paper is in examining the optimal operation of a prototype hydro power plant with electricity prices modelled as a regime switching process. We show that hydro plant value is negatively affected by ramping restrictions, but the extent of the impact depends on key parameters which determine the desirability of frequent changes in water release rates. Interestingly for the case considered, value is not sensitive to ramping restrictions over a large range of restrictions. The results point to the importance of accurately modelling electricity prices in gauging the trade offs involved in imposing restrictions on hydro operators which may hinder their ability to respond to volatile electricity prices and meet peak demands.

JEL Classification: C61, G12, Q25, Q49, Q51, Q58
Keywords: ramping rate, regime switching, hydro power plant, electricity, real options, stochastic control, Hamilton Jacobi Bellman-Partial Differential Equation

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1 Introduction

Hydro electricity is considered an environmentally friendly source of power as it is not associated with the release of carbon and other harmful emissions. Water release rates can be easily adjusted in response to changing electricity demand and prices, making hydro a low cost option for meeting peak demands. However this operating flexibility is known to have environmental costs. As discussed in Edwards et al. (1999), frequent changes in water levels and changing release rates through turbines can alter water temperatures and change the chemical and physical composition of the released water which puts stress on aquatic flora and fauna. There may also be negative effects on beaches and increased erosion of shorelines which is detrimental to the natural balance of the local ecosystem. The extent of these negative environmental impacts is case specific, depending on the size of the particular hydro operation and the fragility of the surrounding ecosystem, including the characteristics of the water body that receives the released water.

There are numerous studies in the biological and environmental sciences literature exploring these negative consequences of hydro operations. There is a consensus that modified water flows are affecting fish and fish habitat, but the response varies widely. Examples of studies documenting these effects include Murchie et al. (2008) and Marty et al. (2009). Niu and Insley (2013) reference numerous other studies which examine the environmental consequences of altered water flows caused by hydro operations. The consequences for the surrounding ecosystem may be judged to be serious enough to warrant restrictions on the management of water flows by a hydro operator. This is an issue that has received attention in numerous jurisdictions across North America. Smokorowski et al. (2009) discuss this issue in relation to the experience in Ontario. Some examples of hydro dams that operate with ramping constraints include the Glen Canyon Dams in Arizona (Veselka et al. (1995) and Harpman (1999)), the Sugar Lake Dam in British Columbia (BC Hydro (2005)), and the Kerr Dam in Montana (Flathead Lakers (2005)).

Hydro operators seeking to maximize profits face a complex dynamic optimization problem. The production of electricity depends in a non-linear fashion on the speed of water released through turbines as well as on the reservoir head, which refers to the height of the water in the reservoir. Releasing water at any given hour reduces the head and hence negatively affects the amount of power that can be produced in the next hour. Eventually the released water will be recovered through water inflow into the reservoir. Maximizing profits over time is thus a balancing act between water inflow to and outflow from the reservoir while attempting to match changing electricity demands over time.

Electricity demand tends to follow a marked daily pattern, peaking during the daytime and early evening hours. It is also subject to seasonal spikes to meet demand for air conditioning in the summer and/or heating in the winter. Electricity cannot be stored and the typical base load power sources including coal and nuclear are much more limited than hydro in their ability to vary generation levels. This results in fairly inelastic supply and consequent volatile electricity prices, particularly in those regions which rely heavily on fossil fuel and nuclear power. Price spikes and jumps are not uncommon. Figure 1 shows the hourly German EEX spot price from January 1 to July 24, 2006. Hydro operators can benefit from this volatility by increasing water release rates (ramping up) in response to high prices and by reducing water release rates (ramping down) in periods of low prices to let water levels recover in the reservoir.

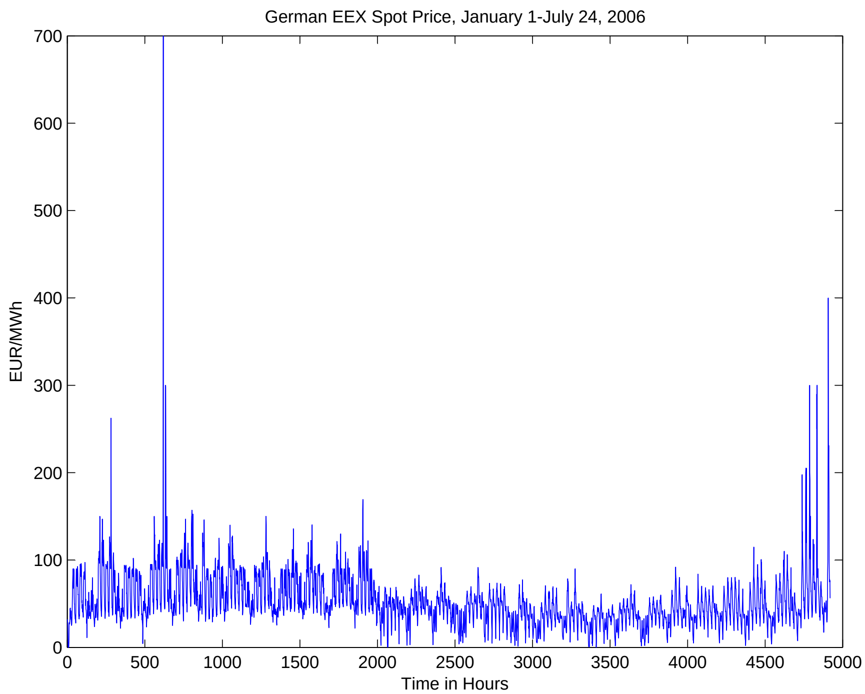


Figure 1: *German EEX Spot Price, January 1-July 24, 2006*

Regulators charged with protecting local ecosystems must consider the consequences of hydro operations for native flora and fish habitat. Restrictions may be imposed on minimum and maximum water levels in reservoirs and rivers, release rates from reservoirs, and the rate of change in the release rate (ramping rate). Any restrictions on hydro operations must be considered in light of the impact on profitability for the hydro operator and on the capability of the electricity grid to meet peak demands. If there is greater reliance on fossil fuels for peaking requirements, there would be added environmental consequences resulting from the harmful emissions from coal, natural gas and petroleum (This issue is addressed in Niu and Insley (2013) in a model of optimal hydro production with deterministic prices.) The optimal choice of restrictions would balance the consequences for the ecosystem with the lost profits to hydro operators and the possible impact on the electricity grid.

Intuitively, ramping restrictions should have the largest impact on profits of hydro operations for which the optimal control without restrictions involves frequent ramping up or down of water release rates. This would be expected in an environment of frequently changing prices in which hydro operators are motivated to adjust water release rates accordingly. It follows that in analyzing the impact of ramping rates it is important to pay particular attention to modelling the dynamics of electricity prices. The jumps and spikes that are observed in electricity prices make electricity price modelling particularly challenging. The recent literature on electricity price modelling, reviewed in Section 2, suggests that regime switching models hold considerable promise.

In the current literature, there are relatively few studies of the costs and associated benefits of restricting ramping rates at hydro power plants. Early work on the economics of ramping restrictions includes Veselka et al. (1995), Edwards et al. (1999), Harpman (1999) and Edwards (2003), who examine the effect of particular ramping rate regimes as environmental constraints, but do not provide extensive analysis of ramping rate restrictions on the power station's optimal operation and value. The trade offs involved in the choice of the optimal ramping rate regime are not addressed in these papers. In a recent study, Niu and Insley (2013) extend these works by considering both the associated benefits and costs of ramping restrictions on hydro profits and on total daily hydro production and the potential implications for other sources of power.

A limitation of these studies is that the optimization models are solved in a deterministic framework. Uncertain electricity prices as well as water inflows imply that optimal hydro power operation is best studied in a stochastic framework. Thompson et al. (2004) and Chen and Forsyth (2008) study the hydro operation and valuation problem in a stochastic optimal control framework using jump diffusion models for electricity prices. Chen and Forsyth (2008) also analyze the impact of ramping restrictions on a hydro plant's value. In a numerical example they find that imposing the most restrictive ramping constraint reduces the value of a hypothetical power plant by 37%. The focus of the Chen and Forsyth (2008) paper is on the efficient solution of the complex stochastic optimization problem. They do not attempt to determine the best model for electricity prices.

This paper contributes to this literature by studying the impact of ramping restrictions on hydro plant operations and value using a regime switching model of electricity prices, which we argue provides a more realistic characterization of electricity prices than other models, such as jump diffusion, used previously in the literature. Hydro operations are modelled as a stochastic control problem which results in a Hamilton-Jacobi-Bellman (HJB) equation. The ramping rate for water releases is chosen to maximize the value of the hydro plant. The optimal control is determined by solving the HJB equation numerically using a fully implicit finite difference approach with semi-Lagrangian time stepping.[1] We examine the impact of a range of ramping restrictions for a medium-sized prototype hydro plant. We focus on how the impact of ramping restrictions depends on the cost of generating hydroelectric power and the characteristics of the assumed price process such as the volatility, probability of regime shift, speed of mean reversion, long run average price and daily seasonality. For comparison we also examine results for a single price regime case (using the base regime in the regime switching model as the single regime) to determine how the presence of two price regimes affects hydro plant value and optimal operations. The parameters of the regime switching model are based on estimates from Janczura and Weron (2009) for German EEX spot prices.

Our study is limited in that we do not address the environmental gains to the aquatic ecosystem, nor the environmental costs of alternate thermal power generation. Specifying these environmental costs and benefits in monetary terms is problematic and beyond the scope of this research. Rather we seek to examine the other side of the equation - the costs of these restrictions in terms of lost profitability. Knowledge of these costs will help illuminate the trade offs involved and inform the design of regulations.

Individual hydro plants are unique in terms of their size, water flows, physical design and linkages to the grid. Optimal hydro operations depend on electricity market structure, demand patterns and reliance on different fuel sources. In this paper we seek to draw some general conclusions about when the effect of ramping restrictions on hydro plant value will be most significant. To preview some of our results, for the medium-sized power plant considered in the analysis, there is a significant effect on value for the most restrictive ramping constraints, but we also observe a range of ramping restrictions over which value is not substantially affected. There are differing effects depending on the characteristics of the assumed price process. We find that ramping restrictions have a larger impact when

[1]See Chen (2008) for a discussion of finite difference schemes based on a semi-Lagrangian method for solving stochastic optimal control problems.

the expected variation in price is increased such as through an increase in volatility which make it desirable to change water release rates relatively frequently. Ramping rates also have a larger impact if the range of likely prices includes values at which the hydro plant is operating at a loss. These and other avenues through which ramping restrictions influence value are explored in depth in this paper.

This paper is organized as follows: in the second section, we provide a brief review of the literature on electricity price modelling; section three describes the modelling of hydro operations; in section four the Hamilton Jacobi Bellman equation for the regime switching case is derived; section five describes the prototype hydro plant used in the empirical study as well as the parameters of the price process; section six contains the empirical analysis of the hydro operation and value under various ramping restrictions, together with a comprehensive sensitivity analysis; we conduct the empirical analysis by including the daily seasonal component in the price process in section seven; lastly, conclusions and directions of future research are given in section eight.

2 Electricity Price Models

There is a rich literature which addresses the challenges of modelling electricity spot prices. At the heart of the modelling issues are the inherent properties of electricity which result in limited transferability over time (storage constraint) and space (transmission constraint). Key characteristics of electricity prices and loads include strong dependence on weather and regular daily, weekly, and monthly patterns. Electricity spot prices tend to exhibit frequent small jumps and occasionally extreme spikes. They show strong mean reversion moving rapidly from price spikes to the mean price, and price level and volatility are highly correlated. Weron (2008) gives a description of these stylized facts.

Weron (2008) provides a good review of the literature on electricity price modelling. When the objective is derivative pricing and risk management, there are two popular classes of stochastic processes for electricity prices: mean-reverting jump diffusion models and mean-reverting Markov regime switching models[2]. Some examples of jump-diffusion models include Deng (2000), Geman and Roncoroni (2006), Escribano et al. (2011), Weron (2008) and Benth

[2]Another class of model is the threshold autoregressive (TAR) model for electricity prices. Recent work includes Rambharat et al. (2005) who incorporate an exogenous variable (temperature) in the proposed TAR model. However, Misiorek et al. (2006) argue that the spot electricity price depends on both fundamentals (such as loads and network constraints) and other psychological and sociological factors which are unquantifiable. They state that "the Markov regime-switching (or simply regime-switching) models, where the regime is determined by an unobservable, latent variable, seem interesting."

et al. (2007). An issue in using jump diffusion models is how to capture the return to a more normal price level after a jump. There are various modelling approaches used in the literature. Chen and Forsyth (2008) in their numerical example model electricity prices with 2 jump processes - a positive jump is followed with high probability by a down jump, bring price back into its normal state. Weron et al. (2004) suggest this represents a good approximation as spikes typically do not last for more than one day.

It is, however, desirable to be able to capture the situation where price suddenly spikes upward and then remains in this spike regime for several periods before returning to a more normal state. This can be observed in the historical data as is shown in Figure 2. This pattern might appear when, for some reason, the grid cannot respond quickly enough to a sudden surge in demand or when there is an unanticipated restriction on supply such as would be caused by grid congestion or an outage in a power station. One way to capture this phenomenon would be to add more jump terms into a jump diffusion model, but this would make the parameter estimation and plant valuation quite complex[3]. Another alternative is the use of Markov regime switching models with base and jump regimes where there is some positive probability of remaining in the jump regime.

Early results on Markov regime switching models include Ethier and Mount (1998) who estimate a two-state switching model with both regimes governed by AR(1) processes. Huisman and De Jong (2003), Weron et al. (2004) and Bierbrauer et al. (2007) make extensions by proposing an independent spike two-regime model using different distributional assumptions for the spike regime. De Jong (2006) modifies the spike regime by using an autoregressive process with Poisson jumps. Further extensions are given by Mount et al. (2006) who allow transition probabilities to depend on the reserve margin and Huisman (2008) who suggests temperature dependent transition probabilities. In two recent studies, Janczura and Weron (2009, 2010) introduce median-shifted spike regime distributions with heteroskedastic dynamics for the base regime. An overview of the development of alternative regime switching models can be found in Janczura and Weron (2010). Some applications of regime switching models in the valuation of physical assets include Chen and Forsyth (2010) for natural gas storage valuation, Heydari and Siddiqui (2010) for valuing a gas-fired power plant and Chen and Insley (2012) for optimal forest harvesting problem.

[3]This requires the estimation of a process with both big and small jumps. When price suddenly spikes upward it would normally be followed by several consecutive up and down jumps at different sizes before jumping back to a normal state. It may be problematic that an up jump could have a long lasting effect if the probability of a down jump is very small. Estimating this process in a jump diffusion model is a nontrivial task. However, this process could be easily generated and estimated in a Markov regime switching model where price follows different dynamics when jumping into a new regime.

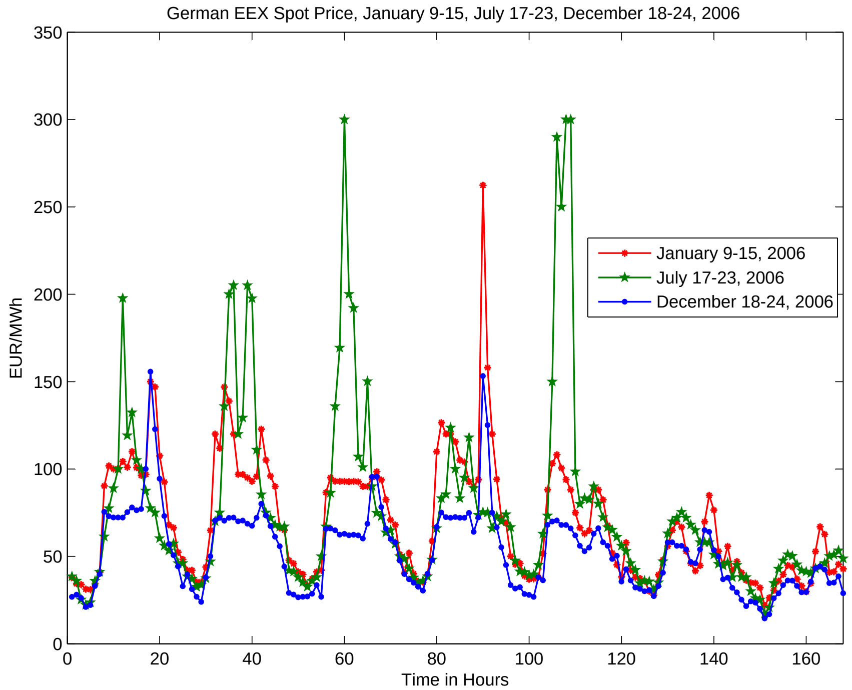


FIGURE 2: *German EEX Spot Price, January 9-15, July 17-23, December 18-24, 2006*

In this paper we adopt a regime switching model based on a statistical estimation done by Janczura and Weron (2009) for German EEX spot prices. The regime switching price model defines N specific regimes and a different price process may be specified for each regime. The transition between regimes is controlled by a continuous-time Markov chain. A general specification of a regime switching process is given in Equation (1) below.

$$dP = \mu^{\imath}(P,t)dt + \sigma^{\imath}(P,t)dZ + \sum_{\jmath=1}^{N} P(\xi^{\imath\jmath} - 1)dX_{\imath\jmath}, \ \imath = 1, ..., N. \tag{1}$$

where

- $\mu^{\imath}(P,t)$ is the generalized drift in regime $\imath$;
- $\sigma^{\imath}(P,t)$ is the generalized volatility in regime $\imath$;
- dZ is the increment of the standard Gauss-Wiener process;
- $\xi^{\imath\jmath}$ is the jump size from state $\imath$ to regime $\jmath$;

- $dX_{\imath\jmath}$ indicates the transition of the Markov chain from regime $\imath$ to regime $\jmath$;

Let $q_{\imath\jmath}$ be the transition intensity of the Markov chain from state $\imath$ to state $\jmath$. Then over an instant dt, we have

$$dX_{\imath\jmath} = \begin{cases} 1 & \text{with probability } q_{\imath\jmath}dt \\ 0 & \text{with probability } 1 - q_{\imath\jmath}dt \end{cases}$$

Over the time interval dt, $q_{\imath\jmath}dt$ is the probability of $dX_{\imath\jmath} = 1$, i.e., the price jumps from P in regime $\imath$ to $P\xi^{\imath\jmath}$ in regime $\jmath$. The number of states N is often chosen arbitrarily for ease of computation, and is set equal to 2 in our empirical study.

Note that in the regime switching model the drift and diffusion terms are specific to the particular regime, indexed by superscript $\imath$. When a regime change occurs, the price jumps from P in regime $\imath$ to $P\xi^{\imath\jmath}$ in regime $\jmath$ and both the drift function and volatility function will change from $\mu^{\imath}(P,t)$ and $\sigma^{\imath}(P,t)$ to $\mu^{\jmath}(P,t)$ and $\sigma^{\jmath}(P,t)$. This contrasts with the common jump diffusion model in which a jump in price does not imply a change in drift and volatility terms. The regime switching model could approximate a jump diffusion model if many different regimes are assumed, and each regime involves a different jump size.

3 Modelling Optimal Hydro Operations

This section presents the model of the hydro power operation used in our analysis. The model describes an individual power plant situated on a dam which controls water levels in a reservoir. The operator is assumed to choose water flows from the dam in order to maximize profits from electricity generation.

3.1 Physical and Environmental Constraints

A typical hydro power plant is operated under certain physical and environmental constraints including minimum and maximum water release rates, maximum and minimum water content in the reservoir, and possibly ramping constraints. We specify similar constraints as in Niu and Insley (2013). The minimum and maximum water release rate requirement can be represented as

$$r^{\min} \leq r \leq r^{\max} \tag{2}$$

where r is the water release rate. Reservoir storage constraints can be stated as

$$w^{\min} \leq w \leq w^{\max} \tag{3}$$

where w represents the water content. The equation of motion for water is governed by the following formula:

$$dw = a(\ell - r)dt \tag{4}$$

where a converts water flow rate to the same unit as the water content. Water inflow rates are stochastic in nature. In order to focus on the stochastic feature of electricity prices and the impact of ramping restrictions on the valuation and optimal operation of the hydro power plant, we assume that the inflow rate ℓ is a positive constant. This simplified assumption is realistic for a short period of time such as the single week used in the empirical analysis in Section 5. An extension of this paper would be to model ℓ by a stochastic differential equation. To prevent the water content falling below $w^{\min}$ or increasing above $w^{\max}$, we follow the approach of Chen and Forsyth (2008) using an augmented equation of motion:

$$dw = H(r, w)a(\ell - r)dt. \tag{5}$$

for any $w \in [w^{\min}, w^{\max}]$ and $r \in [r^{\min}, r^{\max}]$, where H is a smooth function of w and r satisfying

$$\begin{aligned} &H(r, w) \to 0 \text{ if } r > \ell \text{ and } w \to w^{\min}, \\ &H(r, w) \to 0 \text{ if } r < \ell \text{ and } w \to w^{\max}, \\ &H(r, w) = 1 \text{ otherwise.} \end{aligned} \tag{6}$$

In order to satisfy Equation (6) it is assumed that function $H(r, w)$ has the following properties:

$$\begin{aligned} &H(r, w) = O((w - w^{\min})^{\nu}) \text{ if } r > \ell \text{ and } w \to w^{\min}, \\ &H(r, w) = O((w^{\max} - w)^{\nu}) \text{ if } r < \ell \text{ and } w \to w^{\max}, \\ &H(r, w) = 1 \text{ otherwise} \end{aligned} \tag{7}$$

where ν could take any small positive constant value.

The ramping control variable z is defined in the following equation:

$$dr = zdt. \tag{8}$$

The ramping rate z needs to satisfy the following conditions

$$\begin{aligned} &z \geq 0 \text{ if } r = r^{\min}. \\ &z \leq 0 \text{ if } r = r^{\max}. \end{aligned} \tag{9}$$

to prevent the water release rate from increasing above the maximum releasing rate or decreasing below the minimum releasing rate. As in Chen and Forsyth (2008), in order to satisfy conditions in (9) we will require that

$$\begin{aligned} &\min\{z\} = O((r - r^{\min})^{\theta}) \text{ if } r \to r^{\min}, \\ &\max\{z\} = O((r^{\max} - r)^{\theta}) \text{ if } r \to r^{\max}, \end{aligned} \tag{10}$$

where θ could take any small positive constant value. In addition, the up-ramping and down-ramping constraints can be expressed as

$$dr \leq r^u dt. \tag{11}$$

$$-dr \leq r^d dt. \tag{12}$$

where r^u and r^d represent the maximum allowed up-ramping and down-ramping rates respectively. We rewrite these two equations as

$$-r^d dt \leq dr \leq r^u dt. \tag{13}$$

which is

$$-r^d \leq z \leq r^u. \tag{14}$$

Let $Z(r)$ denote the set of admissible controls that satisfy constraint (14) and conditions (10). This gives $Z(r) \subseteq [z^{\min}, z^{\max}]$.

3.2 The Optimization Problem

The present value of net revenue from power generation from $t = t_1$ to $t = T$ is given by the following equation

$$\int_{t_1}^{T} e^{-\rho t} q(r, h(w))(P - c) dt. \tag{15}$$

where ρ is the discount rate, q is the amount power produced which is a function of the water release rate r and the head h, and c is the unit cost of hydro power production, which is assumed to be a positive constant. Therefore, $q(r, h(w))(P - c)$ is the instantaneous profit for the hydro power plant. The objective is to maximize Equation (15) subject to the set of physical and environmental constraints for a hydro power plant described in Section 3.1. Let $V^\imath(P, w, r, t_1)$ denote the value of the hydro plant under the optimal control in regime $\imath$ under the risk adjusted (or risk neutral) measure.

$$V^\imath(P, w, r, t_1) = \max_z E^Q \left[\int_{t_1}^{T} e^{-\rho(t-t_1)} H(r, w) q(r, h(w))(P - c) dt | P(t) = \tilde{P}, w(t) = \tilde{w}, r(t) = \tilde{r} \right]. \tag{16}$$

where $\tilde{P}$, $\tilde{w}$, and $\tilde{r}$ represent current values for $P(t)$, $w(t)$ and $r(t)$, respectively, and the maximization is subject to

$$Z(r) \subseteq [z^{\min}, z^{\max}]. \tag{17}$$

$$dw = H(r, w) a(\ell - r) dt. \tag{18}$$

$$dr = z dt. \tag{19}$$

$$dP = \mu^\imath(P, t) dt + \sigma^\imath(P, t) dZ + \sum_{\jmath=1}^{N} P(\xi^{\imath\jmath} - 1) dX_{\imath\jmath}. \tag{20}$$

where H is a penalty function on profits given by Equation (6), so that the profit decreases to zero as $r > \ell$ and $w \to w^{\min}$ or $r < \ell$ and $w \to w^{\max}$, and otherwise remains at $H(r, w) q(r, h(w))(P - c)$. $Z(r)$ specifies the set of admissible controls. In addition, the electricity price, P, follows Equation (20) which is assumed to be a risk adjusted specification. The transition equations governing water balance and water release are specified in Equations (18) and (19) respectively. Note that constraints (2) and (13) are incorporated into (17) and constraint (3) is incorporated into (16) and (18).

4 Hamilton Jacobi Bellman Equations

4.1 HJB-PDE for the Regime Switching Model

Following Kennedy (2007), we use the standard hedging approach in options pricing to derive the HJB-PDE (Hamilton Jacobi Bellman Partial Differential Equation) for the value of the hydro power station.[4] Under the risk adjusted measure[5] the value of the hydro plant in regime $\imath$ satisfies the following equation

$$
\begin{aligned}
\bar{r}V^\imath = \sup_{z \in Z(r)} (z\frac{\partial V^\imath}{\partial r}) + H(r,w)a(\ell - r)\frac{\partial V^\imath}{\partial w} + \frac{1}{2}(\sigma^\imath)^2(P,t)\frac{\partial^2 V^\imath}{\partial P^2} + (\mu^\imath(P,t) - \Lambda^\imath \sigma^\imath(P,t))\frac{\partial V^\imath}{\partial P} \\
+ H(r,w)q(r,h(w))(P-c) + \frac{\partial V^\imath}{\partial t} + \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda^Q_{\imath\jmath}(V^\jmath - V^\imath).
\end{aligned}
\tag{21}
$$

where $\bar{r}$ is the risk free interest rate, $\Lambda^\imath$ is the market price of risk in state $\imath$ and $\lambda^Q_{\imath\jmath}$ is the risk-neutral transition intensity from state $\imath$ to $\jmath$ ($\jmath \neq \imath$). The electricity price given by Equation (1) is used to derive Equation (21).

The HJB-PDE can be interpreted as a no-arbitrage condition. In the risk neutral world the return on the asset must equal the risk free rate, $\bar{r}$. On the right hand side of the equation we see all the sources of return on the asset which, from left to right, include changes due to the optimal choice of the ramping rate, z; changes due to water inflows and releases which affect the water content, w; changes due to first and second order effects of electricity price variation, P; cash flow from electricity sales; changes in V with time; and lastly the effect on value of the risk of a price regime change.

The domain for Equation (21) is $(P, w, r) \in [0, \infty] \times [w^{\min}, w^{\max}] \times [r^{\min}, r^{\max}]$. The numerical solution is computed in a finite domain: $(P, w, r) \in [0, P^{\max}] \times [w^{\min}, w^{\max}] \times [r^{\min}, r^{\max}]$. Equation (21) is solved backwards in time from $t = T$ to $t = t_1$. It is convenient to define $\tau = T - t$ as the time remaining in the life of the asset. Equation (21) can be

[4] More details can be found in an appendix available from the authors.

[5] The hydro plant valuation with the regime switching model for electricity prices is associated with the regime switching risk, therefore the market is not complete. This indicates that there is no unique equivalent martingale measure that allows us to move from the physical measure to the risk adjusted measure. Forsyth and Vetzal (2014) discuss that one way to pin down the measure for pricing is to use an expanded set of hedging instruments. Then the risk-neutral transition intensity could be uniquely determined by the prices of these instruments. Our derivation for the HJB-PDE follows this method.

rewritten in terms of τ as follows:

$$\frac{\partial V^\imath}{\partial \tau} = CV^\imath + BV^\imath + \sup_{z \in Z(r)} (z \frac{\partial V^\imath}{\partial r}) + H(r,w)a(\ell - r)\frac{\partial V^\imath}{\partial w} + H(r,w)q(r,h(w))(P-c). \quad (22)$$

where the operators C and B are given by

$$CV^\imath = \frac{1}{2}(\sigma^\imath)^2(P,t)\frac{\partial^2 V^\imath}{\partial P^2} + (\mu^\imath(P,t) - \Lambda^\imath \sigma^\imath(P,t))\frac{\partial V^\imath}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda_{\imath\jmath}^{Q})V^\imath.$$

$$BV^\imath = \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda_{\imath\jmath}^{Q} V^\jmath.$$

We specify a functional form of the electricity price as in Chen and Forsyth (2008) for both the drift and diffusion terms and assume $\mu^\imath(P,t) = \alpha^\imath(K^\imath - P), \sigma^\imath(P,t) = \sigma^\imath P$. Now $CV^\imath$ is written as

$$CV^\imath = \frac{1}{2}(\sigma^\imath)^2 P^2 \frac{\partial^2 V^\imath}{\partial P^2} + [\alpha^\imath(K^\imath - P) - \Lambda^\imath \sigma^\imath P]\frac{\partial V^\imath}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1 \\ \jmath \neq \imath}}^{N} \lambda_{\imath\jmath}^{Q})V^\imath.$$

A numerical approach is required to solve this non-linear HJB-PDE. We will use a fully implicit finite difference scheme with semi-Lagrangian time stepping. This numerical approach converges to the viscosity solution of the HJB equation as is shown in Chen and Forsyth (2007, 2008).[6]

4.2 Boundary Conditions

Boundary conditions are required to fully specify the optimization problem. For the terminal boundary condition at $t = T$ (i.e. $\tau = 0$), we assume the value of the asset is zero, which is a common assumption in the literature. (See Thompson et al. (2004) for example.)

$$V^\imath(P, w, r, \tau = 0) = 0. \quad (23)$$

It can be shown that the solution of Equation (22) does not require information outside of the domains of w and r and hence no special boundary conditions are required. At

[6] Details on the numerical algorithms for solving Equation (22) are given in an appendix available from the authors.

$w^{\min}, w^{\max}, r^{\min}, r^{\max}$, we simply solve the PDE along the corresponding boundaries.[7] However, the computational domain of the price could differ for any two different regimes.

Taking the limit of the PDE equation as $P\to 0$, we obtain the boundary PDE for $\imath = 1, 2, \ldots, N$.

$$\frac{\partial V^\imath}{\partial \tau} = C_0 V^\imath + BV^\imath + \sup_{z\in Z(r)} (z\frac{\partial V^\imath}{\partial r}) + H(r,w)a(\ell - r)\frac{\partial V^\imath}{\partial w} - H(r,w)q(r,h(w))c; P\to 0 \quad (24)$$

where the operator B is the same as above and C_0 is given by

$$C_0 V^\imath = \alpha^\imath K^\imath \frac{\partial V^\imath}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1\\ \jmath\neq\imath}}^{N} \lambda_{\imath\jmath}^{Q})V^\imath.$$

For $P\to\infty$, we apply the commonly used boundary condition $V^\imath_{PP} = 0$ (Wilmott (1998)), which implies that

$$V^\imath \simeq x(w,r,\tau)P + y(w,r,\tau).$$

The boundary PDE now is written as

$$\frac{\partial V^\imath}{\partial \tau} = C_1 V^\imath + BV^\imath + \sup_{z\in Z(r)} (z\frac{\partial V^\imath}{\partial r}) + H(r,w)a(\ell - r)\frac{\partial V^\imath}{\partial w} + H(r,w)q(r,h(w))(P-c); P = P^{\max} \quad (25)$$

where the operators C_1 is given by

$$C_1 V^\imath = [\alpha^\imath (K^\imath - P) - \Lambda^\imath \sigma^\imath P]\frac{\partial V^\imath}{\partial P} - (\bar{r} + \sum_{\substack{\jmath=1\\ \jmath\neq\imath}}^{N} \lambda_{\imath\jmath}^{Q})V^\imath.$$

We solve the PDE in the region $P \in (0, P^{\max})$. At the boundaries we solve PDE (24) at $P = 0$ and PDE (25) at $P = P^{\max}$.

[7] See Chen and Forsyth (2008) for more details.

5 Specification of the Empirical Example

5.1 Description of the Hydro Plant

The empirical analysis is done for a generic medium-sized dam. For reasons of data availability, the physical details of the dam, in terms of reservoir capacity and water flow, are based on the Abitibi Canyon generating station in north eastern Ontario. However, the optimal ramping choices are studied in relation to German EEX electricity prices. We note dams of a size similar to Abitibi Canyon are also found in Europe.[8]

The analysis is conducted for a time horizon of one week[9] which is a sufficient time scale for the questions addressed in this paper. We abstract from the natural variations in water inflow and assume a constant inflow[10] of 6,671 CFS, which is the average hourly inflow rate based on data for the historical water inflow for the hydro station from January 1, 2001 to November 30, 2006. As noted earlier this assumption is considered realistic for the short time horizon we are using. A stochastic mean reverting diffusion water inflow process could be incorporated into this empirical study, which however would add another dimension to the HJB-PDE. The cost of generating hydroelectric power is assumed constant at 20 EUR/MWh for the hydro station.

We use a standard hydro power production function from the power engineering literature. The amount of electricity produced can be represented by

$$q(r, w) \propto r \times h(r, w) \times e(r, h). \tag{26}$$

where q is the power output, r is the water release rate, h is the gross head, w is the water content, e is the efficiency factor and $\propto$ means proportion. Under some simplifying assumptions about its functional form from Niu and Insley (2013), the following production

[8] For example the Donzère-Mondragon Dam in France has an installed capacity of 354 MW. The combined physical capacity (CPC) of the generators at the Abitibi Canyon Station is about 19 thousand Cubic-feet-per-second (CFS) and the storage capacity of the reservoir is about 17 thousand acre-feet. This station has a generation capacity of 336 MW. Detailed descriptions of the generating station can be found in Statistics Canada (2000), Hendry and Chang (2001), and *http://www.opg.com/power/hydro/northeast_plant_group/abitibi.asp*.

[9] In two related studies, the time horizon is 15 days in Thompson et al. (2004) and one week in Chen and Forsyth (2008). For the empirical studies in this paper, the computational time varies with the level of ramping restrictions and takes from less than a day to one week.

[10] This only represents the average case in our sample period. If data were available, the analysis for a specific season (wet or dry period) could be easily conducted by calculating the average hourly inflow rate in that period.

Table 1: *Parameter Values for the Prototype Hydro Station*

Parameter	Value	Parameter	Value
ℓ	6671 CFS	$w^{\max}$	17000 acre-feet
g	32.15 feet/square-second	$w^{\min}$	7000 acre-feet
ρ	1000 kg/cubic-meter	$r^{\max}$	15000 CFS
b	0.0089	$r^{\min}$	2000 CFS
e	0.87	$q^{\max}$	336 MW
CPC	19000 CFS	$q^{\min}$	0 MW
–	–	r^u	3000 CFS-hr
–	–	r^d	3000 CFS-hr

function is used in our empirical analysis

$$\begin{aligned} q(r, h(w)) &= 0.001 \times g \times r \times h(w) \times e \\ &= 0.28 \times r \times h(w) \end{aligned} \tag{27}$$

where g is the gravitational constant (32.15 feet-per-square-second) and the factor 0.001 converts q to MW from KW, r is in CFS and h is in feet. The efficiency factor is assumed constant at 0.87. We use a linear functional form between the head and water content in the reservoir, represented by

$$h(w) = b \times w \tag{28}$$

where w is the water content in acre-feet and b is assumed to be 0.0089.

In the empirical analysis, the operational and environmental constraints and their associated values for the base case are specified as

- Up-ramping and down-ramping constraints are 3,000 Cubic-feet-per-second per hour (CFS-hr);
- The minimum water release requirement is 2,000 CFS and the maximum release constraint is 15,000 CFS;
- The minimum water content requirement is 7,000 acre-feet and the maximum value is 17,000 acre-feet;

These are the cases examined in Niu and Insley (2013). The input parameters for the hydro power production and hydro dam specifications are reported in Table 1.

5.2 Specification of the Regime Switching Price Process

We base our Markov regime switching model on the model estimated in Janczura and Weron (2009), which is parsimonious and can capture pertinent characteristics for electricity price dynamics. They use the German EEX spot price from 2001-2009 to estimate the following model[11]

$$dP = \eta(\mu_1 - P)dt + \sigma_1\sqrt{P}dZ. \tag{29}$$

$$\log(P - m) \sim N(\mu_2, \sigma_2^2),\ P > m. \tag{30}$$

where the base regime is the CIR (Cox-Ingersoll-Ross) process and the spike regime is the shifted lognormal distribution (with higher mean and variance than those in the base regime) which assigns zero probability to prices below the median m. Introducing heteroskedasticity through the CIR square root process reflects the fact that electricity price volatility generally increases with price level, because positive price shocks tend to increase volatility more than negative shocks. The shifted spike regime distribution (lognormal) is used in order to correctly separate spikes from the 'normal' price behavior. Their empirical study shows that these specifications are more realistic for electricity spot prices and lead to better spike identification and goodness-of-fit than in the standard mean-reverting Markov regime switching models (i.e. the Vasicek or Ornstein-Uhlenbeck process) with constant volatility dynamics.

Janczura and Weron (2009) obtain maximum likelihood estimates of the parameters of Equations (29) and (30) using the algorithm of Kim (1994). The mean daily EEX spot price data is first deseasonalized by removing a long run seasonal trend representing changing climate and consumptions conditions during the year and long-term non-periodic structural changes. In addition a weekly periodic component is removed. Table 2 shows the parameter estimates obtained from Janczura and Weron (2009) for the 2001 to 2005 period of their sample[12]. Note that since the estimation is done using daily data, the parameters can be interpreted using a daily time scale.

The model specified in Equations (29) and (30) is different than that shown in Equation (1). In particular the spike regime is not modelled as an Itô process. We adapt the Janczura

[11] Janczura and Weron (2009) estimate the Markov regime switching models with shifted lognormal or Pareto spikes and Vasicek or CIR base regimes. The test results for the base regime show that the CIR specification is more suitable than the Vasicek specification and the p-values for the spike regime indicate that the lognormal distribution gives a better fit compared to the Pareto spike distribution.

[12] Janczura and Weron (2009) provide the parameter estimates for two subsamples: January 1, 2001 - January 2, 2005 and January 3, 2005 - January 3, 2009. The results indicate that prices in the first, less volatile period give a better fit.

Table 2: *Parameter Values Estimated by Janczura and Weron (2009)*

Parameter	Value
μ_1	47.194 EUR/MWh
μ_2	3.44
η	0.36
σ_1	0.73485
σ_2	0.83066
m	46.54 EUR/MWh
λ_{12}	0.0089
λ_{21}	0.8402

Base regime: $dP = \eta(\mu_1 - P)dt + \sigma_1\sqrt{P}dZ$
Spike regime: $log(P - m) \sim N(\mu_2, \sigma_2^2),\ P > m.$

and Weron (2009) model to conform to standard Itô processes in both regimes so that it can be incorporated into the HJB equation describing optimal hydro dam operation. Specifically it is assumed the regime switching process can be specified as follows for the base and spike regimes respectively:

$$dP = \eta(\mu_1 - P)dt + \sigma_1\sqrt{P}dZ + P(\xi^{12} - 1)dX_{12}. \tag{31}$$

$$dP = \sigma_2(P - m)dz + P(\xi^{21} - 1)dX_{21},\ P > m. \tag{32}$$

The P-measure transition intensities are as defined in Equation (1).

Note that Equations (31) and (32) also include terms at the end which specify the jump in price level when a regime change happens. The spike regime is only defined for prices above the median, m. It was thus necessary to assume that when a change in regime happens from the base to the spike regime there will be a jump up in price level, with the jump size reflecting the ratio of the long run expected values in the base and spike regimes. When a transition from the spike to the base regime occurs, a jump down in price will occur at level of equal but opposite magnitude.

The parameters in Table 2 are estimated under the P measure (physical measure). To derive the option value using no arbitrage arguments, we require the market price of risk to convert the P measure to the Q measure. Under the Q measure Equations (31) and (32) become

$$dP = [\eta(\mu_1 - P) - \Lambda_1\sigma_1\sqrt{P}]dt + \sigma_1\sqrt{P}d\hat{Z} + P(\xi^{12} - 1)d\hat{X}_{12}. \tag{33}$$

$$dP = \sigma_2(P - m)d\hat{z} + P(\xi^{21} - 1)d\hat{X}_{21},\ P > m. \tag{34}$$

where $d\hat{Z}$ and $d\hat{z}$ are the increment of the standard Gauss-Wiener processes under the Q

TABLE 3: *Parameter Values for the Regime Switching Model*

Parameter	Value	Parameter	Value
μ_1	47.194 EUR/MWh	η	0.36
m	46.54 EUR/MWh	c	20 EUR/MWh
σ_1	0.73485	σ_2	0.83066
ξ^{12}	1.6470	ξ^{21}	0.6072
λ_{12}^{Q}	0.0089	λ_{21}^{Q}	0.8402
Λ_1	-0.2481	-	-
$P_1^{\max}$	200 EUR/MWh	$P_2^{\max}$	200 EUR/MWh
$P_1^{\min}$	0 EUR/MWh	$P_2^{\min}$	48 EUR/MWh
T	168h	$\bar{r}$	0.05 annually

Base regime: $dP = [\eta(\mu_1 - P) - \Lambda_1\sigma_1\sqrt{P}]dt + \sigma_1\sqrt{P}d\hat{Z} + P(\xi^{12} - 1)d\hat{X}_{12}$.
Spike regime: $dP = \sigma_2(P - m)d\hat{z} + P(\xi^{21} - 1)d\hat{X}_{21},\ P > m$.

measure, and Λ_1 is the market price of risk which adjusts the drift term in the base regime from the P to the Q measure. $d\hat{X}_{12}$ and $d\hat{X}_{21}$ indicate the transition of the Markov chain under the Q measure. In this empirical study we rely on existing studies of the market price of risk to convert the drift from the physical measure to the risk neutral measure. However, as there are no current studies converting P-measure transition probabilities to Q-measure probabilities[13], sensitivity analysis is employed to investigate the impact of changing transition probabilities.

The parameter values adopted for Equations (33) and (34) are shown in Table 3. The long run price, μ_1, median price, m, mean reversion speed, η, and volatilities σ_1 and σ_2 are taken from the Janczura and Weron estimates given in Table 2. Janczura and Weron (2009) follow the common practice in the statistical analysis of electricity price of using daily average log-prices, instead of hourly prices, to estimate the chosen models, partially because they are better suited to certain distributions and the estimated parameters have robust statistical properties. However, this is problematic for understanding optimal decisions in power plant operations, which are made on an hourly or half hourly basis based on current spot electricity prices and demand. In addition, the use of daily average prices ignores any regular daily patterns, common in electricity prices. In this paper we undertake sensitivity analysis to investigate the impact of changing parameter estimates including imposing a daily seasonal pattern to the price process.

As noted above ξ^{12} is set equal to the ratio of the spike regime mean price to the base regime mean price. ξ^{21} is roughly the inverse of ξ^{12}. So, on average $\xi^{21} \times \xi^{12}$ is around 1

[13]Q-measure probabilities could be calibrated to the observed prices of traded options for electricity prices. However this is beyond the scope of the current paper.

to avoid arbitrage opportunities. λ_{12}^Q and λ_{21}^Q represent the regime transition intensities in the risk neutral world. The risk adjusted probability of switching from regime $\imath$ to regime $\jmath$ is $\lambda_{\imath\jmath}^Q dt$. As expected the probability of remaining in the base regime over a single day is very high, so that the spike regime occurs quite rarely. As was noted earlier, these estimated transition probabilities[14] from Janczura and Weron (2009) are under the P-measure and we use them as the risk neutral transition probabilities in our empirical study, which is problematic. However, in the sensitivity analysis we will check the robustness of our results by assuming different transition probabilities.

Another important parameter is the market price of risk Λ, representing the excess return over the risk-free rate that the market expects for taking extra risks. The empirical evidence for the market price of risk in the power markets are mixed in terms of the magnitude, variability and sign which could be positive or negative. Estimated values generally vary over time and across markets. Some of the empirical estimations are provided by Lucia and Schwartz (2002), Cartea and Figueroa (2005), Kolos and Ronn (2008) and Weron (2008). In Weron (2008) the market prices of risk implied from Nord Pool's AEO-GB0300 Asian-style options and GB0300 futures range from -0.2 to 0.1 during the period November 1, 1999 - January 28, 2000. In a much longer period (February 2, 1998 - May 16, 2001), he observes for most of the time the market price of risk is negative and ranges from -0.5 to 0.1. This negative market price of risk is consistent with the observations from some previous studies for the Nord Pool and U.S. markets (Weron (2008)). In this paper we assume the market price of risk is a deterministic constant and use the estimated value from Cartea and Figueroa (2005), where they use historical spot data and forward data from the England and Wales electricity market[15] and find the market price of risk is -0.2481. The negative market price of risk implies relatively high futures prices compared to spot prices, which is consistent with a product where storage options are very limited and purchasing futures contracts is the only way to assure access to electricity in the future for a set price. With few storage options Weron (2008) notes that the negative value might be explained by "a higher incentive for hedging on the demand side relative to the supply side, because of the non-storability of

[14]In the literature, constant transition probabilities are usually estimated and used for derivative pricing and risk management. However, this specification may not be appropriate in our asset pricing model as the transition probability for electricity prices may not be constant, but depend on some observable variables such as the demand level.

[15]The market price of risk would be specific to a particular market. Ideally, we should use the estimated market price of risk based on the spot and futures price data from the German EEX Power market. However, since the continental European and British electricity grids are inter-connected via the HVDC Cross-Channel link and BritNed we expect the market price of risk would be similar in these two regions.

electricity as compared to the (limited and costly but still existent) storage capabilities of fuel" (page 1112).

Table 3 also shows the assumed variable cost of electricity production, c, the upper and lower limits on price in the numerical analysis, $P_\imath^{\min}$ and $P_\imath^{\max}$, $\imath = 1, 2$, the timeframe for the analysis in hours, T, and the risk free rate, $\bar{r}$.

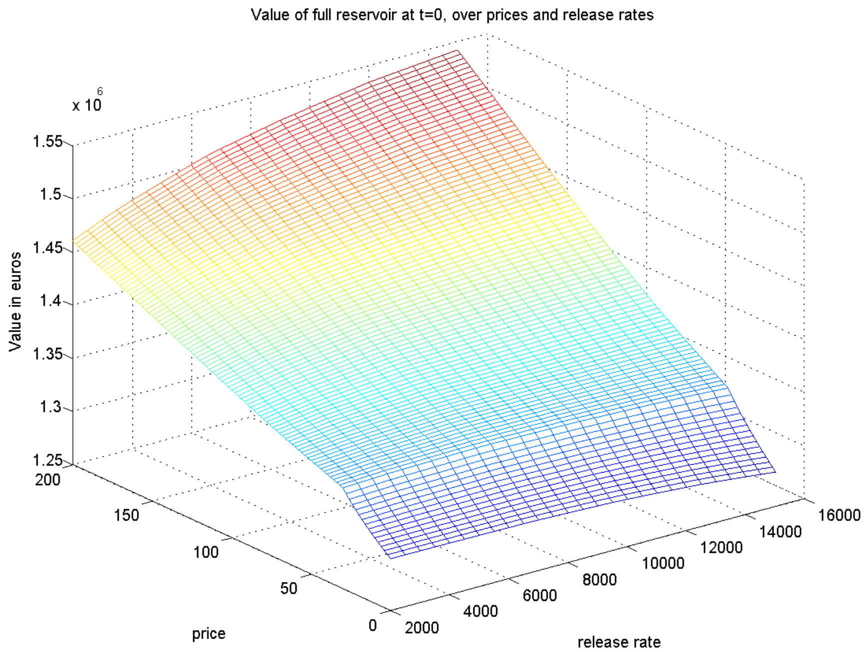
6 Empirical Results

6.1 Base Case

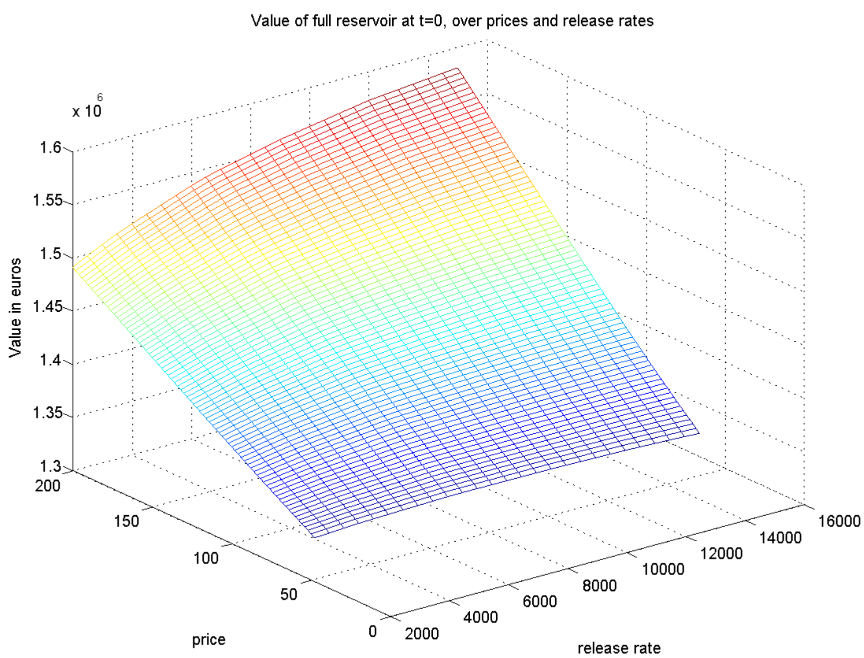
In this section, we present the numerical results for the base case with ramping constraints at 3,000 CFS-hr. The value of the hydro power station is a function of electricity price, water content, release rate, time and regime. We are interested in how these variables will affect the value and operation of the power plant under various levels of ramping restrictions. Due to the high dimensionality of the value function, we can only analyze the results by looking at various combinations of the variables while holding other variables constant. So, given any ramping restrictions, for the numerical results we will focus on the following relationships: power plant value verse price and release rate; optimal ramping rate verse price and time; optimal ramping rate verse price and reservoir level.

Figures 3(a) through 5(b) show the results for this regime switching case when the constraints for ramping rate, water release rate and water content are imposed. Figures 3(a) and 3(b) plot the value of the hydro power plant as a function of water release rate and spot price at time zero for the base regime and the spike regime respectively when the reservoir level is at the full capacity and the inflow rate is at 6,671 CFS. Note that the spike regime is only defined for prices above 46.54 EUR/MWh while the base regime is defined over the complete range shown from 0 to 200 EUR/MWh. In the base regime we observe that the value of the hydro power plant is increasing with the spot price. However, the hydro power plant's value is increasing with the water release rate for prices above the marginal cost of electricity generation of 20 EUR/MWh and decreasing with the water release rate for prices below the marginal cost of electricity generation. This is due to the fact that the power plant incurs losses by generating power when the spot price moves below the marginal cost. But in the spike regime the value of the hydro power plant is increasing with both water release rate and spot price. Not surprisingly, at any given water release rate and spot price, the value of the hydro power plant in the spike regime is higher than its value in the base regime. More specifically, in the base regime the value reaches the maximum of EUR 1,541,400 when the current release rate is 15,000 CFS and the price is 200 EUR/MWh and its value reaches the minimum of EUR 1,282,100 when the current release rate is 15,000 CFS and the price is 0 EUR/MWh. Even at a price of 0 EUR/MWh the power plant's value is still positive due to the fact that the price will move up at a later time and generate positive profits. Similarly, in the spike regime the value reaches the maximum of EUR 1,580,300 when the current release rate is 15,000 CFS and the price is 200 EUR/MWh and its value reaches the minimum of EUR 1,340,900 when the current release rate is 2,000 CFS and the price is 48 EUR/MWh.

Figures 4(a) and 4(b) plot the optimal operational strategies for the hydro power plant as a function of time and spot price for the base regime and the spike regime respectively when the reservoir level is at the full capacity, the water release rate is at 8,500 CFS which is midway between the upper and lower limits. These diagrams allow a better view of the impact of price on the optimal strategy and also indicate the effect of imposing a terminal time of one week. For the base regime we can clearly observe that at lower prices it is optimal to ramp down and preserve the water in the reservoir, whereas for high prices it is optimal to ramp up thereby increasing the release rate and power generation. For a typical hour there are four operational regions: the power plant will ramp down at the maximum limit of 3,000 CFS-hr in order to generate less power and keep the reservoir at full capacity if the price is below 20 EUR/MWh; if the price lies between 20 EUR/MWh and 40 EUR/MWh the station will ramp down at 2,000 CFS-hr; when the price is between 40 EUR/MWh and 60 EUR/MWh the station will ramp down at 1,500 CFS-hr; as the price moves above 60 EUR/MWh the station will ramp up at the maximum limit of 3,000 CFS-hr. It may also be observed that the optimal control strategy remains largely unchanged over time, but behaves quite differently when the time moves close to the terminal time. Since the terminal boundary condition of zero value is imposed, the power plant will ramp up at the maximum allowed rate when time is close to the end of the time horizon. For the spike regime the station will mostly ramp up at the maximum limit of 3,000 CFS-hr due to higher prices.

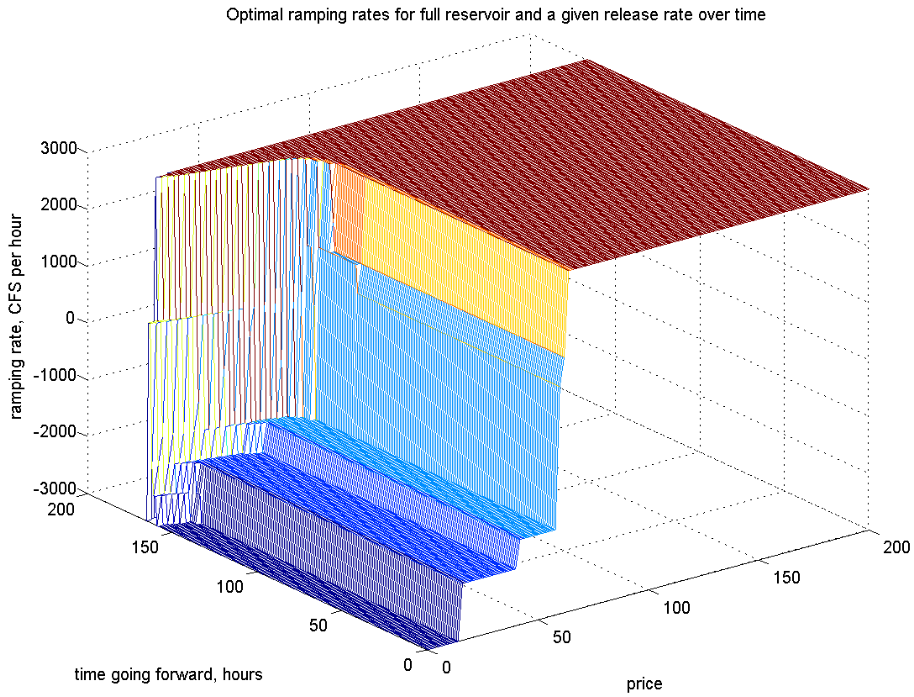


(a) *Value over Price and Release Rate for the Base Regime*

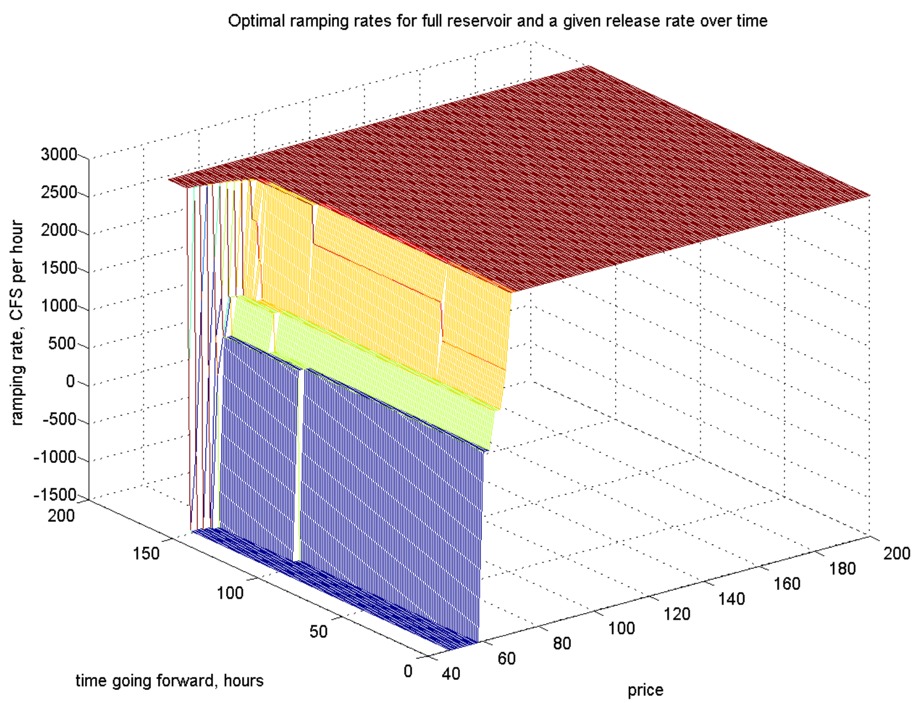


(b) *Value over Price and Release Rate for the Spike Regime*

FIGURE 3: *Value over Price and Release Rate*

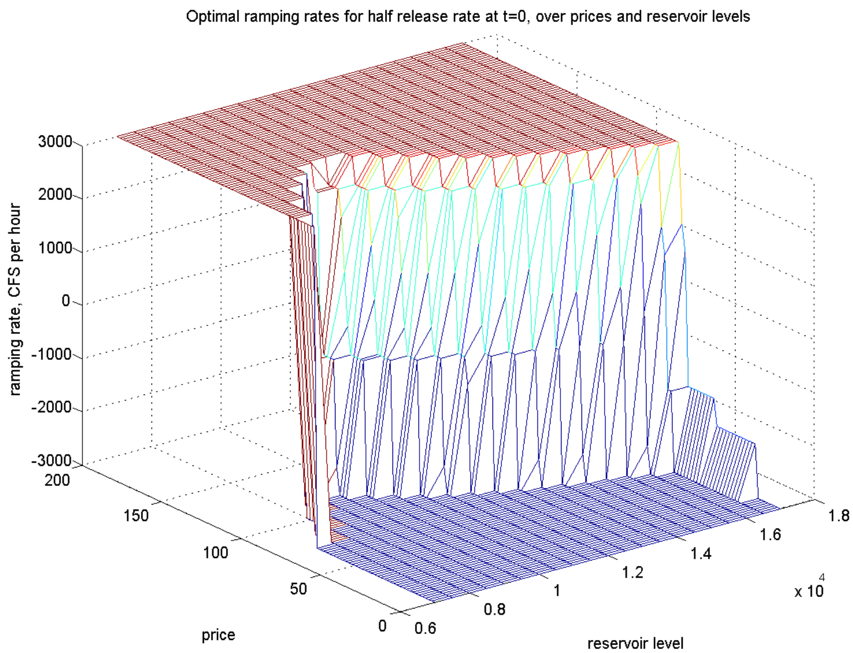


(a) *Optimal Ramping Rate over Price and Time for the Base Regime*

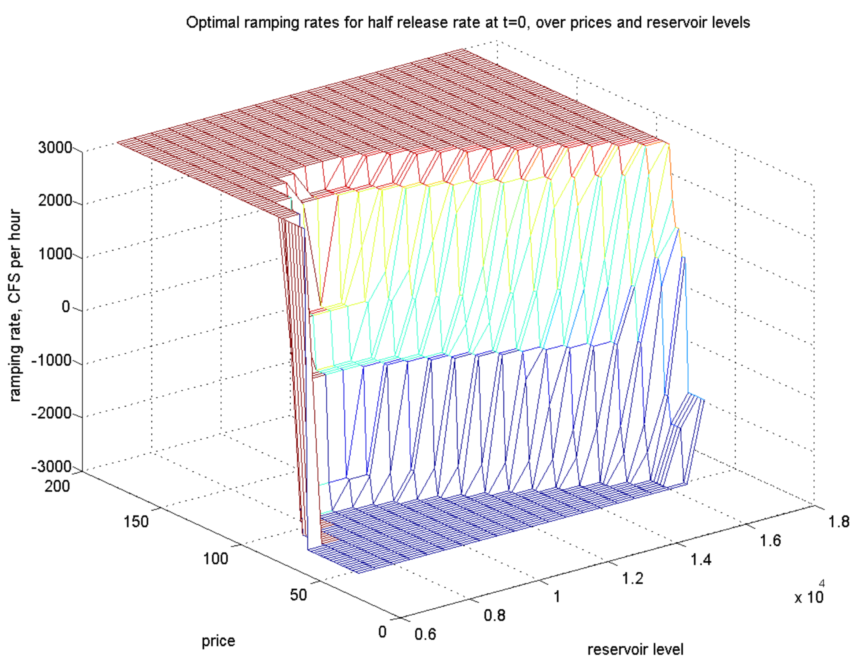


(b) *Optimal Ramping Rate over Price and Time for the Spike Regime*

FIGURE 4: *Optimal Ramping Rate over Price and Time*



(a) *Optimal Ramping Rate over Price and Reservoir Level for the Base Regime*



(b) *Optimal Ramping Rate over Price and Reservoir Level for the Spike Regime*

FIGURE 5: *Optimal Ramping Rate over Price and Reservoir Level*

Figures 5(a) and 5(b) show the optimal control strategies of the hydro power plant as a function of reservoir level and spot price at time zero for the base regime and the spike regime respectively when the release rate is at 8,500 CFS. In general, it is optimal to ramp down at the maximum rate of 3,000 CFS-hr when prices are low and ramp up at the maximum of 3,000 CFS-hr when prices are high. Recall that ramping up increases the water release rate which generates more electricity, but also reduces the potential for power generation in the next period by reducing water content and reservoir head. The price boundary which divides these optimal actions can be seen in the figures to vary with reservoir level. We will refer to the prices at this boundary as critical prices. Ignoring the points at the upper and lower reservoir boundaries, it will be observed that critical prices generally decrease as the reservoir level is increased. This reflects the fact that additional water content is less valuable when the reservoir is already quite full. At lower water levels the operator is more inclined to let the water levels recover through down ramping.

The price boundary which divides the up-ramping and down-ramping regions is observed to change near the reservoir boundaries of 7,000 acre-feet and 17,000 acre-feet in Figure 5(a). At the upper reservoir boundary, the critical price declines starkly and the optimal ramping rates fall to a level between the upper and lower limits. It makes sense that when the dam is nearly full, the operator will ramp down at this intermediate rate, which prevents water content from going over the limit, while still keeping the reservoir as full as possible.

At the lower reservoir boundary, the critical price falls, meaning that at this boundary the region over which up-ramping is optimal becomes wider. This is not sustainable as the minimum water content limit would be violated. However, it is never optimal to operate anywhere close to the lower reservoir boundary since the power plant's value is positively related to the water content.

6.2 Impact of Changing Ramping Restrictions

In this section, we consider the effect of ramping constraints on the value of the hydro plant. Values for different ramping constraints for full reservoir and at maximum and half of the maximum release rates are shown in Table 4.[16] Figure 6 plots the results for the full release rate. In general, the more restrictive the constraint the greater is the limitation on the station's operational flexibility and the larger the impact on value. However for this particular example, value is not highly sensitive to ramping rate restrictions. The maximum impact

[16]Note that we refer to Table 4 as the benchmark case for the impact of ramping restrictions. We refer to 3000 CFS-hr in that table as the base case.

Table 4: *Numerical Results Under Various Ramping Restrictions: Benchmark Case*

Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
Case		No Ramping Restrictions	5000 (CFS-hr)	3000(CFS-hr)(Base Case)	1000 (CFS-hr)	250 (CFS-hr)
HF	Value	1368900	1364000	1355500	1339800	1310700
	%ch	N/A	-0.4	-1.0	-2.1	-4.3
FF	Value	1367600	1361400	1350700	1318000	1254100
	%ch	N/A	-0.5	-1.2	-3.6	-8.3
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1401100	1395300	1385900	1367500	1337200
	%ch	N/A	-0.4	-1.1	-2.4	-4.6
FF	Value	1403600	1397700	1387700	1358000	1298800
	%ch	N/A	-0.4	-1.1	-3.2	-7.5
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1517300	1509700	1497400	1467700	1428500
	%ch	N/A	-0.5	-1.3	-3.3	-5.9
FF	Value	1529600	1524100	1514900	1490300	1449000
	%ch	N/A	-0.4	-1.0	-2.6	-5.3

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

of the most restrictive scenario with ramping limits of 250 CFS-hr is a reduction of 8.3% in the base regime compared to no ramping restrictions. It may also be observed from the table that the impact is smaller (-4.3% compared to -8.3%) when the current release rate is already at half the maximum value. This is because, starting from a lower release rate, less adjustment is needed in the release rate when ramping down in response to low prices. However these results are specific to the case under consideration. In the next section, several sensitivities are presented to investigate how the impact of ramping restrictions is affected by key parameter values.

6.3 Sensitivity Analysis

Sensitivity analyses were undertaken for five key parameters of interest: the speed of mean reversion in the base regime, the transition intensities in each regime, the volatilities, the long run mean in the base regime, and the cost of generating hydroelectric power. A sixth sensitivity is conducted to determine the significance of including the spike regime. In this case the base regime is modelled as a single regime. A summary of the results of all the sensitivities is provided in Figures 7(a) and 7(b). Detailed results are provided in Appendix A, Tables 6 through 12.

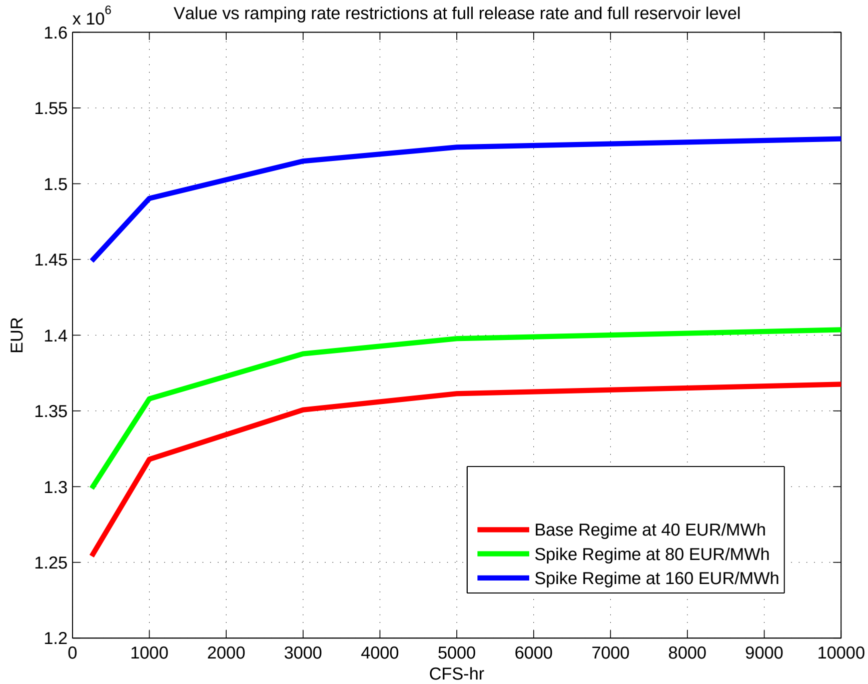


FIGURE 6: *Value vs Ramping Rate Restrictions at Full Release Rate and Full Reservoir Level*

- **Mean reversion speed, η.** Doubling the speed of mean reversion from 0.36 to 0.72 results in a decline in project value in the order of 6 to 7% for comparable cases at all levels of ramping restrictions. For example at 3000 CFS-hr ramping restrictions for the full release rate, full reservior (FF) case, value falls from EUR 1.352 million (Table 4) to EUR 1.276 million (Table 6 in the Appendix). Ramping rate restrictions have a similar impact on value in this high η case as in the benchmark case, i.e., value falls proportionately more as ramping restrictions are increased when ramping rates are already quite restrictive. However the impact of increasing ramping rates is slightly smaller. At the most restrictive ramping constraints of 250 CFS-hr the value drops by 7.6% in the worst scenario (base regime, FF, $P_0 = 40$ EUR/MWh) compared to the case of mean reverting rate at 0.36, where value decreases by 8.3% in the worst scenario. The higher η implies that price stays closer to the long run mean, which provides less opportunity for the hydro plant owner to benefit from an upward movement in price by ramping up and selling more power. Since the motivation to ramp up (or down) will be reduced, ramping restrictions would be expected to have a smaller impact. The percentage change in value for the most restrictive ramping constraints compared to the benchmark is shown in Figure 7. (See Appendix A, Table 6.)

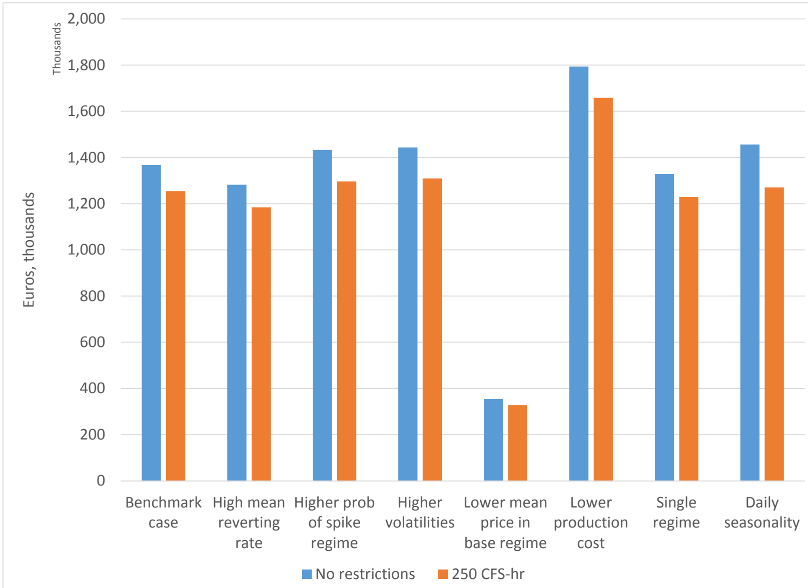
- **Regime transition probabilities, $\boldsymbol{\lambda}_{ij}^{Q}$.** We consider the impact of a higher probability that price resides in the spike regime by increasing λ_{12}^{Q} from 0.0089 to 0.02 and decreasing λ_{21}^{Q} from 0.8402 to 0.7402 while keeping η at 0.36. Again, we observe similar patterns for the impact of ramping restrictions in these two cases. But with the new transition probabilities the value is uniformly higher and the ramping impact is slightly larger (but of the same order of magnitude). At the most restrictive ramping constraints of 250 CFS-hr value drops by 9.5% in the worst scenario (base regime, FF, $P_0 = 40$ EUR/MWh) compared to the benchmark case where value decreases by 8.3% in the worst scenario. The change in transition probabilities implies more time is spent in the spike regime, which as expected increases the value of the power station. But with a greater likelihood of being in the spike regime, the imposition of ramping restrictions limiting the ability to respond to higher prices has a larger impact. (See Appendix A, Table 7.)

- **Volatilities, $\boldsymbol{\sigma_j}$.** In the base regime the volatility, σ_1 is increased from 0.74 to 0.93. In the spike regime σ_2 is increased from 0.83 to 1.43. The other parameters are kept at benchmark case values. The same patterns for the impact of ramping restrictions emerge, but the higher volatilities cause two major changes. First, the value of the hydro plant is uniformly higher. This is consistent with the standard result on options pricing, i.e. higher volatility is associated with higher options value. Second, for higher volatilities the impact of ramping restrictions on the power plant's value is slightly larger (but of the same order of magnitude). At the most restrictive ramping constraints of 250 CFS-hr value drops by 9.3% in the worst scenario (base regime, FF, $P_0 = 40$ EUR/MWh) compared to the case of low volatilities, where value decreases by 8.3% in the worst scenario. This is caused by the fact that a higher volatility results in more frequent high prices and low prices. So these restrictive ramping constraints affect the plant's ability to capture profits by ramping up or down quickly in response to more frequent price changes. (See Appendix A, Table 8.)

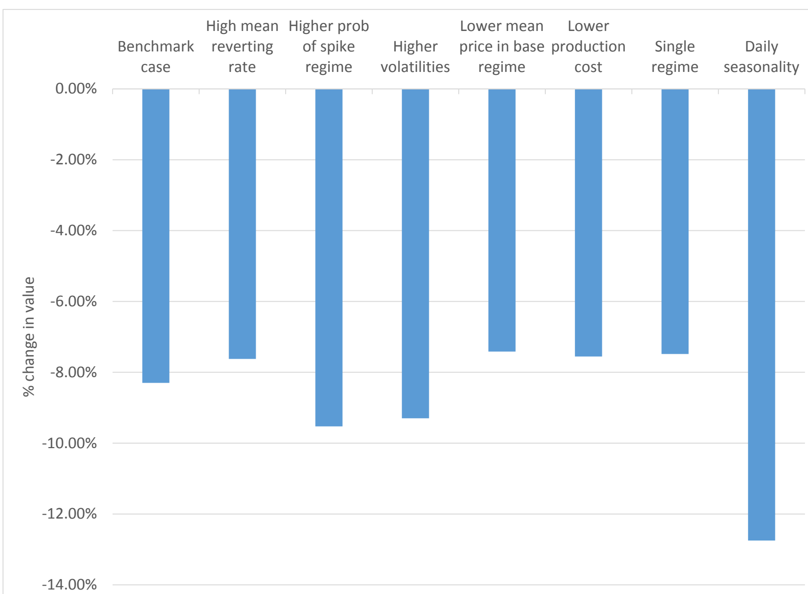
- **Long run mean price, $\boldsymbol{\mu_1}$.** In this sensitivity μ_1 is decreased from 47.194 EUR/MWh to 27.194 EUR/MWh. Compared to the benchmark case, there is a similar pattern for the impact of ramping restrictions. However, a 20 EUR/MWh reduction of the base regime mean price results in a significantly lower value for the power plant. The impact of ramping restrictions on the power plant's value compared to the benchmark case is different depending on the initial release rate. Starting at a release rate of half the maximum value, ramping restrictions have a larger impact in this case compared to the benchmark. However when the release rate is at its full value, restrictive ramping constraints cause a smaller percentage reduction in value compared to the benchmark case. Figures 7(a) and 7(b) show the comparison for the full release rate in the base regime where the reduction in value at the most restrictive ramping constraints of 250 CFS-hr is 7.4% compared to 8.3%. The impact of ramping restrictions is dwarfed by the overall decline in value caused by the lower long run price. More details are given in Appendix A, Table 9.

- **Cost of hydro generation, c.** The parameter c is halved from 20 EUR/MWh to 10 EUR/MWh. For this lower cost of production, the value of the hydro plant is significantly higher, and the impact of increasing ramping rates is slightly smaller. At the most restrictive ramping constraints of 250 CFS-hr value drops by 7.6% in the worst scenario (base regime, FF, P_0 = 40 EUR/MWh) compared to the case with cost at 20 EUR/MWh, where value decreases by 8.3% in the worst scenario. A lower production cost means a higher net profit and higher power plant value. Since this lower cost also reduces the likelihood of negative profit, ramping restrictions would be expected to have a smaller impact. (See Appendix A, Table 10.)

- **Single regime.** This sensitivity is done to determine the importance of the spike regime to the value and optimal operations of the hydro plant. As expected, the single regime case gives an overall lower value than the two regime case. However the results are surprisingly close, differing only by 2 to 3 percent. In addition in the single regime case, ramping restrictions have a smaller impact than in the two regime case. However the change is again quite small, with the most severe restrictions reducing value by 7.5 % in the single regime case (base regime, FF, P_0 = 40 EUR/MWh) compared to 8.3 % in the two regime benchmark case. Restrictive ramping constraints will limit the power plant's ability to make profits during price spikes, therefore the power plant's value is more severely affected when the spike regime is included. However, price spikes are still rare events. In addition if the hydro plant is already operating at full capacity when a spike occurs, no ramping is needed for the plant to capture the extra profits from the price spike. (See Appendix A, Table 12.)



(a) *Hydro plant value for no ramping restrictions compared to 250 CFS-hr for various cases, Base Regime, FF, P= 40 EUR/MWh*



(b) *Percent change in value for 250 CFS-hr ramping restriction versus no ramping restriction, Base Regime, FF, P= 40 EUR/MWh*

FIGURE 7: *Summary of Sensitivity Analysis, Base Regime, FF, P= 40 EUR/MWh*

The results of these six sensitivities show that ramping rates will have a larger effect when the price process is such that frequent ramping up or down is desirable - such as with higher volatilities or more frequent price spikes. A reduced effect of ramping rate restrictions is observed when price is higher relative to the cost of generation, as in the case when the cost of generation is reduced. In the empirical examples in this paper, price is mostly above the variable cost of producing power. This is why ramping restrictions do not have a huge effect on plant value even at the most restrictive level. If the price process were such that price fell below cost frequently, then we expect that ramping restrictions would have a much more noticeable effect since the hydro operator would be unable to quickly respond to the low price by ramping down in order to decrease power supply.[17]

These and other additional numerical experiments[18] show that the ramping effect is largely determined by the level of the price relative to the cost of generation, i.e., how long and how frequent the price is close to or lower than the cost of hydro power, and is less sensitive to the price jumps. In another words, in Janczura and Weron (2009) where the electricity price is almost surely higher than the cost of power generation, the results show that the effect of ramping restrictions on the power plant's ability to benefit from the spike regime is minimal. This is because the hydro operation will be optimally operating somewhere close to the maximum release rate most of the time. Even though the price jumps frequently, it is still a rare event compared to the base regime. In Janczura and Weron (2009) the unconditional probability of the spike regime is as low as 0.0104.

7 Including Daily Seasonality

As noted in the introduction, regular daily patterns are typical in many electricity markets, with prices rising during the time of peak daily demand. The empirical study above is based on the Markov regime switching model of Janczura and Weron (2009), where any regular daily trends are ignored. In this section a representative daily seasonal pattern is imposed on the base case price model in order to gain some insight into the impact of a regular daily

[17] An additional sensitivity analysis is conducted for the market price of risk which is increased from -0.2481 to 0.2481. The impact of ramping restrictions is almost the same as the benchmark case. However, a positive market price of risk results in a significantly lower value for the power plant since the market requires a higher return for taking extra risks. Additionally, for a positive market price of risk the impact of ramping restrictions on the power plant's value is very similar to the benchmark case with a negative market price of risk.

[18] Niu (2014) empirically investigates the impact of ramping rate restrictions on hydro plant operations and profitability using a regime switching model with multiple jump sizes.

pattern (daily seasonality) on optimal operations and the impact of ramping constraints.

Under the Q measure, it is assumed the regime switching model with the daily seasonality can specified as follows:

$$dP = [\eta(\mu(t) - P) - \Lambda_1\sigma_1\sqrt{P}]dt + \sigma_1\sqrt{P}d\hat{Z} + P(\xi^{12} - 1)d\hat{X}_{12}. \quad (35)$$

$$\mu(t) = \mu_1 + \phi \sin\Big(\frac{2\pi(t - t_0)}{24}\Big). \quad (36)$$

$$dP = \sigma_2(P - m)d\hat{z} + P(\xi^{21} - 1)d\hat{X}_{21},\ P > m. \quad (37)$$

where

- $\mu(t)$ is the long-term equilibrium price with the daily price cycle;
- μ_1 is the equilibrium price without the daily price fluctuation;
- ϕ is the daily price trend;
- t_0 is the time of the daily peak of the equilibrium price;

The daily seasonal component (Equation (36)) for the base regime (Equation (35)) is taken from Chen and Forsyth (2008) and Equation (37) is the same as Equation (34) for the spike regime. In Chen and Forsyth (2008), ϕ and t_0 are set at 15 and 7.7π respectively. The parameter value for μ_1 is shown in Table 3. Figure 8 shows simulations of the price process based on these parameter values (Table 3). The three realizations of the price process look not unreasonable compared to the hourly German EEX spot price as is shown in Figure 2, and we believe this case can provide some useful intuition. Clearly a complete analysis of the impact of ramping restrictions in any particular case would require the estimation of the relevant price process on an hourly basis so that regular daily patterns are captured empirically.

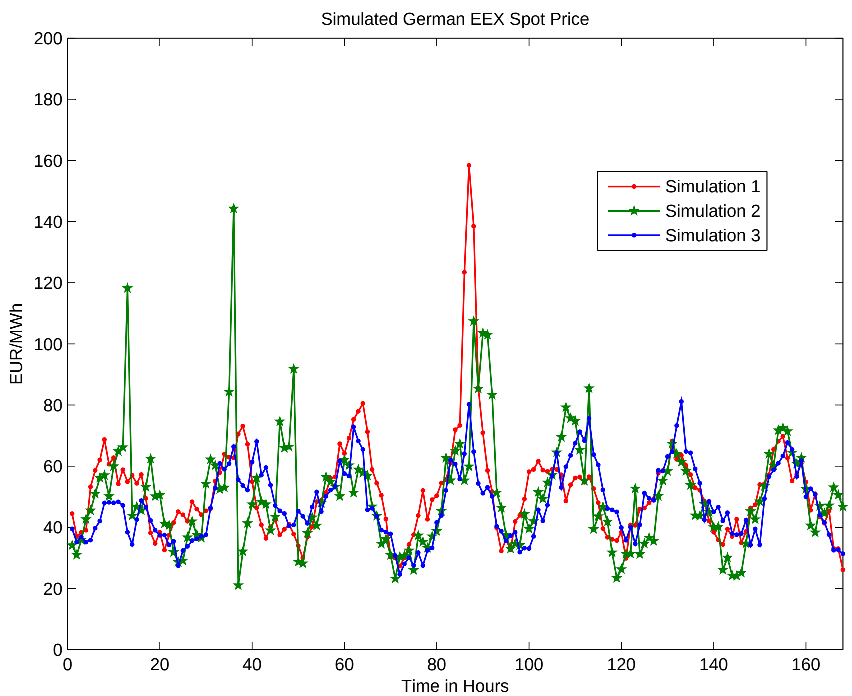
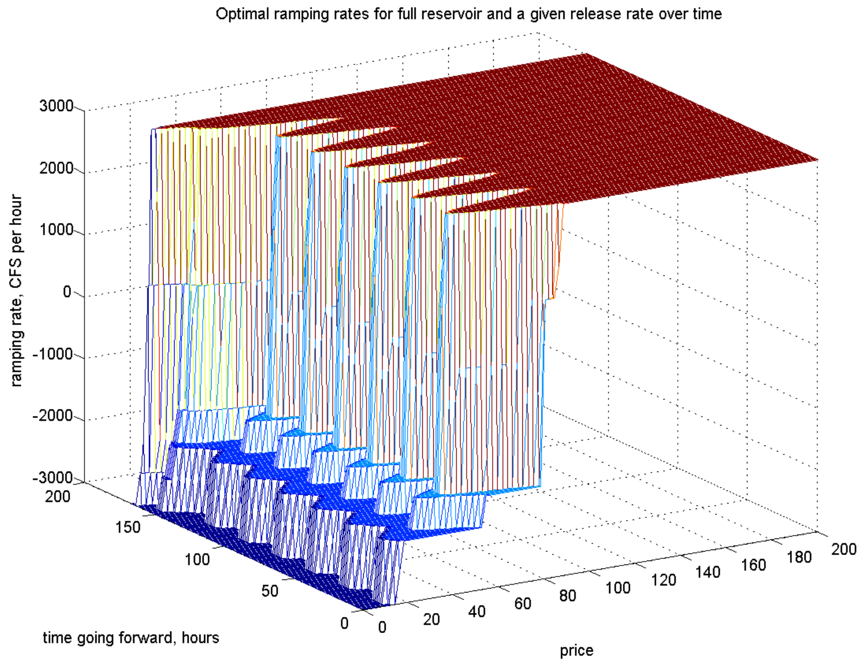
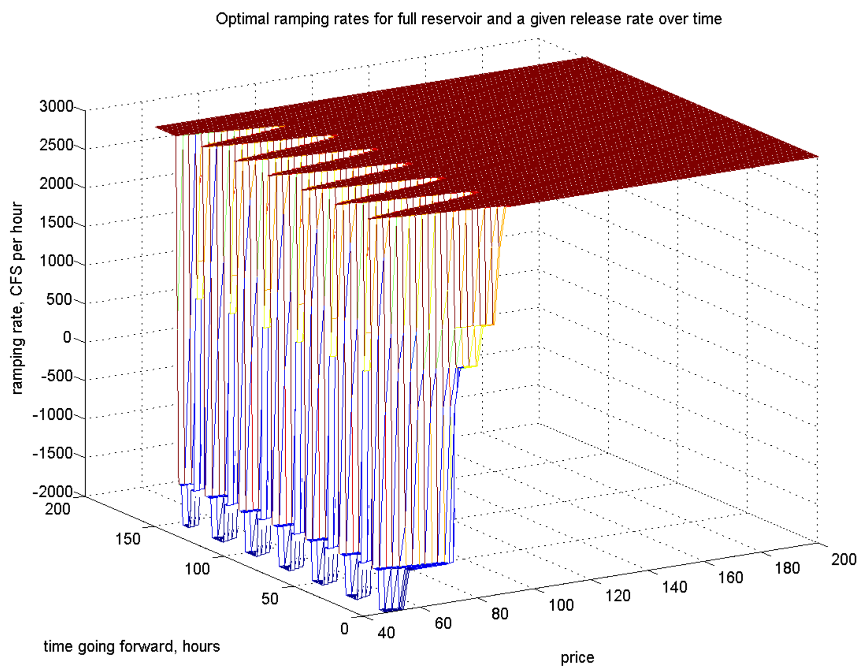


FIGURE 8: *Simulated German EEX Spot Price*



(a) *Optimal Ramping Rate over Price and Time for the Base Regime (Including the Daily Seasonality)*



(b) *Optimal Ramping Rate over Price and Time for the Spike Regime (Including the Daily Seasonality)*

FIGURE 9: *Optimal Ramping Rate over Price and Time (Including the Daily Seasonality)*

With this daily seasonal component included, the value of the hydro plant is uniformly higher than the value for the base case. This is mainly due to the fact that the seasonal component results in several consecutive hours of high prices on a daily basis which increases the opportunity to earn higher profits. The differences in value compared to the base case can be seen in Figure 7(a) for the case with no ramping restrictions, as well as the case with ramping limits of 250 CFS-hr.

There are some changes in the optimal operational strategy shown in Figures 9(a) and 9(b), which plot the optimal ramping rate as a function of time and spot price. These figures may be compared to the base case shown in Figures 4(a) and 4(b). With daily seasonality the optimal control strategy repeats each day. For a particular day, when the price lies approximately[19] between 32 EUR/MWh and 62 EUR/MWh the optimal control follows the pattern of the daily price movement very closely, i.e. ramping up when the price moves up and ramping down when the price goes down. If we truncate 9(a) for prices below 48 EUR/MWh, the optimal control strategies are basically the same as those for the spike regime as shown in 9(b). The optimal control strategies of the hydro power plant as a function of reservoir level and spot price for the case with the daily seasonality (not shown) are similar to those depicted in Figures 5(a) and 5(b) for the benchmark case.

The impact of ramping constraints on value are shown in Table 5. Figure 10 provides a comparison of the seasonality case with the benchmark case for two price levels when the reservoir is full. The major difference is that at the more restrictive constraint levels, the impact of ramping restrictions is greater with seasonality than in the benchmark case. With the most severe restrictions of 250 CFS-hr value drops by 12.8% in the worst scenario compared to the benchmark case where value decreases by 8.3% in the worst scenario. Including the seasonal component the power plant also faces several consecutive hours of low prices at a daily basis. In this case, the power plant is not able to ramp down quickly during the period of low prices because of these restrictive ramping constraints. Therefore the ramping impact tends to be larger.

It may also be observed from Figure 10 that the difference in hydro plant value is reduced between the two cases for more restrictive ramping constraints. This reflects the inability of the hydro owner to respond to the daily fluctuations under ramping constraints. This numerical experiment provides further evidence that ramping restrictions have a larger impact when the expected variation in price is increased such as through the extra seasonal

[19] It can be seen that $-1 \leq \sin\left(\frac{2\pi(t-t_0)}{24}\right) \leq 1$, therefore the long-term equilibrium price with the daily price cycle $\mu(t)$ satisfies $32.194 \leq \mu(t) \leq 62.194$.

component which makes it desirable to change water release rates relatively frequently.

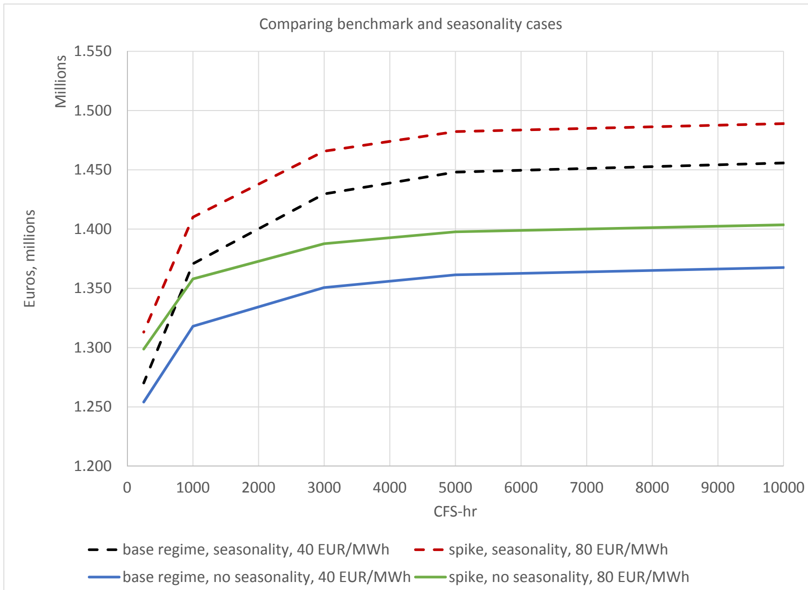


FIGURE 10: *Comparing the Impact of Ramping Restrictions with and without Daily Seasonality, Dashed Lines Show Cases with Seasonality*

8 Conclusions

The ability of hydro facilities to respond quickly through ramping to changing demand and price conditions is one of the benefits of hydro power. However the significant negative consequences of ramping on aquatic ecosystems needs to be considered by regulators. These negative impacts are case specific, dependent on the physical structure of the dam and the ecological conditions of particular rivers and streams. In cases where ramping rate restrictions are being considered, apart from the environmental gains to the river ecosystem, there should be a recognition of the costs imposed on hydro operators in terms of lost profits as well as potential environmental impacts that result from the need to utilize alternative sources of electricity. Ideally ramping rate regulations would be determined through a careful analysis of all the potential impacts. This paper contributes to our understanding of the impact of ramping restrictions on the hydro station's operation and value.

Table 5: *Numerical Results with Daily Seasonality*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	1458000	1452900	1439700	1392700	1320300
	%ch	N/A	-0.4	-1.3	-4.5	-9.4
FF	Value	1455800	1448100	1429700	1370800	1270200
	%ch	N/A	-0.5	-1.8	-5.8	-12.8
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1487600	1481900	1468000	1419800	1346000
	%ch	N/A	-0.4	-1.3	-4.6	-9.5
FF	Value	1489000	1482300	1465700	1410100	1313100
	%ch	N/A	-0.5	-1.6	-5.3	-11.8
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1602700	1595600	1579900	1524400	1438400
	%ch	N/A	-0.5	-1.4	-4.9	-10.3
FF	Value	1613100	1607500	1593800	1545300	1463200
	%ch	N/A	-0.4	-1.2	-4.2	-9.3

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

The accurate modelling of electricity prices is still a topic of considerable debate in the literature. Markov regime-switching models are becoming more popular as a means of capturing in a parsimonious manner the major characteristics of electricity prices such as price spikes. This paper contributes to the literature by demonstrating the effect of using a regime switching model to analyze ramping rate issues in a prototype hydro power plant.

For a prototype hydro dam we valued the power plant and modelled the lost value for a range of ramping restrictions for the regime switching model over a one week period. In most scenarios the optimal control is of a "bang-bang" type, i.e., ramping up at the upper limit when prices are high and ramping down at the upper limit when prices are low. The exception to this is when the dam is up against one of the other constraints such as maximum/minimum release rates or maximum water content in the dam. We find that hydro plant value is significantly affected (in the order of 8%) in the case of the most severe ramping constraints. However we also find a range of less severe ramping constraints for which value is impacted by less than 3%. By using the base regime in the regime switching model as the single regime, the results show that introducing the spike regime increases the value of the power plant and also increases the impact of ramping restrictions, but in our empirical example this impact is not very sensitive to the spike regime. In addition, when a daily seasonal component is included in the price process, with the most severe restrictions value drops by 12.8% compared to the benchmark case where value decreases by 8.3%.

All of these results will depend on the specifics of the particular hydro plant under consideration such as the physical structure as well as the market structure that the plant operates in such as the pricing system. Nevertheless, the sensitivity analysis shows that these empirical findings are quite robust for this regime switching model for electricity prices. Notwithstanding that price spikes have a significant effect on the value of the hydro power plant, one conclusion is that the ramping effect on value mostly depends on the level of the price relative to the cost of generation (how long and how frequent the price is close to or below the cost), but is not very sensitive to the price jumps. In general ramping restrictions have a larger effect in an environment where frequent ramping up or down is desired. A lower speed of mean reversion, higher volatility, more frequent transition to the spike regime and extra daily seasonal component all have the effect of increasing the impact of ramping restrictions.

An important conclusion of the paper is that ramping restrictions should not be determined in isolation, but rather using a cost-benefit approach that evaluates the trade offs involved. This paper has identified some of the important trade offs that should be examined more carefully in future research. These include the impact on hydro plant's operation and value.

There are several directions for further research. First, we could account for uncertainty in water inflow assuming it could be modelled as mean reverting stochastic processes, but this would add another dimension to the HJB-PDE and obtaining the numerical solution for this problem is nontrivial. Second, this paper adapted electricity process parameter estimates found in the literature. Further work is needed in estimating an electricity price model using hourly data, preferably in the Q-measure to avoid the necessity of determining an appropriate market price of risk. Third, further efforts are needed to construct a measure of the environmental benefits for the river ecosystem gained by imposing these ramping restrictions. Finally, the ramping issue could be better studied through a partial equilibrium model of hydro-thermal competition. Then we could analyze the impact of ramping restrictions on the electricity price, production, transmission, associated pollutant emission of thermal power and social welfare.

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APPENDIX

A Tables for the Sensitivity Analyses

This Appendix presents detailed tables of results that support the discussion in Section 6.3.

Table 6: *Numerical Results with Different Mean Reverting Rate*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	1282900	1278600	1271400	1258800	1233000
	%ch	N/A	-0.3	-0.9	-1.9	-3.9
FF	Value	1281700	1276400	1267300	1240400	1184000
	%ch	N/A	-0.4	-1.1	-3.2	-7.6
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1310000	1304800	1296400	1281000	1254000
	%ch	N/A	-0.4	-1.0	-2.2	-4.3
FF	Value	1312900	1307700	1299000	1273400	1220000
	%ch	N/A	-0.4	-1.1	-3.0	-7.1
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1398200	1391000	1379500	1353800	1319100
	%ch	N/A	-0.5	-1.3	-3.2	-5.7
FF	Value	1412300	1407400	1399000	1373900	1329500
	%ch	N/A	-0.4	-0.9	-2.7	-5.9

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

Table 7: *Numerical Results with Different Transition Probabilities*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	1434100	1427400	1416900	1394100	1357400
	%ch	N/A	-0.5	-1.2	-2.8	-5.4
FF	Value	1432700	1424700	1411700	1371000	1296200
	%ch	N/A	-0.6	-1.5	-4.3	-9.5
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1471000	1463500	1451900	1426100	1387800
	%ch	N/A	-0.5	-1.3	-3.1	-5.7
FF	Value	1473400	1465800	1453600	1416000	1346800
	%ch	N/A	-0.5	-1.3	-3.9	-8.6
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1602100	1592700	1578200	1540400	1491500
	%ch	N/A	-0.6	-1.5	-3.9	-6.9
FF	Value	1614000	1606800	1595500	1563400	1514600
	%ch	N/A	-0.5	-1.2	-3.1	-6.2

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

Table 8: *Numerical Results with Different Volatilities*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	1444500	1437900	1427400	1405900	1370700
	%ch	N/A	-0.5	-1.2	-2.7	-5.1
FF	Value	1443100	1435200	1422100	1382600	1308900
	%ch	N/A	-0.6	-1.5	-4.2	-9.3
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1491500	1484100	1472600	1447300	1408800
	%ch	N/A	-0.5	-1.3	-3.0	-5.6
FF	Value	1494000	1486400	1474200	1437600	1370700
	%ch	N/A	-0.5	-1.3	-3.8	-8.3
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1641500	1632400	1618100	1580500	1531000
	%ch	N/A	-0.6	-1.4	-3.7	-6.7
FF	Value	1653100	1646300	1635800	1608600	1565400
	%ch	N/A	-0.4	-1.1	-2.7	-5.3

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

Table 9: *Numerical Results with Different Base Regime Mean*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	353240	349390	344550	335130	325670
	%ch	N/A	-1.1	-2.5	-5.1	-7.8
FF	Value	354010	350210	345400	333960	327760
	%ch	N/A	-1.1	-2.4	-5.7	-7.4
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	403830	399100	392940	378800	364600
	%ch	N/A	-1.2	-2.7	-6.2	-9.7
FF	Value	409700	405900	401180	390370	386960
	%ch	N/A	-0.9	-2.1	-4.7	-5.6
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	540470	533690	524440	497970	470150
	%ch	N/A	-1.3	-3.0	-7.9	-13.0
FF	Value	557720	554080	549920	542920	542270
	%ch	N/A	-0.7	-1.4	-2.7	-2.8

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

Table 10: *Numerical Results with Different Production Cost*

Value and Change of Value in Regime 1 at Time 0 When the Initial Price is 40 EUR/MWh						
Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
HF	Value	1794800	1790600	1782100	1765500	1729300
	%ch	N/A	-0.2	-0.7	-1.6	-3.7
FF	Value	1793400	1787700	1776500	1738400	1657900
	%ch	N/A	-0.3	-0.9	-3.1	-7.6
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1826200	1821200	1811700	1792600	1755700
	%ch	N/A	-0.3	-0.8	-1.8	-3.9
FF	Value	1828500	1823200	1812800	1778000	1702600
	%ch	N/A	-0.3	-0.9	-2.8	-6.9
Value and Change of Value in Regime 2 at Time 0 When the Initial Price is 160 EUR/MWh						
HF	Value	1939800	1932900	1920600	1890600	1846000
	%ch	N/A	-0.4	-1.0	-2.5	-4.8
FF	Value	1951500	1946600	1937200	1908200	1852800
	%ch	N/A	-0.3	-0.7	-2.2	-5.1

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.

Table 11: *Summary of Results from the Sensitivity Analysis*

Case	Power Plant Value	Ramping Impact on Power Plant Value
Base Regime		
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (Benchmark)	N/A	N/A
η=0.72, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (I)	Lower ($\star$)	Smaller ($\star$)
η=0.36, λ_{12}^{Q}=0.02, λ_{21}^{Q}=0.7402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (II)	Higher ($\star$)	Larger ($\star$)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.93, σ_2=1.43, μ_1=47.194, c=20 (III)	Higher ($\star$)	Larger ($\star$)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=27.194, c=20 (IV)	Lower ($\star$)	Larger ($\star$) (Most Cases)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=10 (V)	Higher ($\star$)	Smaller ($\star$)
Spike Regime		
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (Benchmark)	N/A	N/A
η=0.72, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (I)	Lower ($*$)	Smaller ($*$) (Most Cases)
η=0.36, λ_{12}^{Q}=0.02, λ_{21}^{Q}=0.7402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=20 (II)	Higher ($*$)	Larger ($*$)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.93, σ_2=1.43, μ_1=47.194, c=20 (III)	Higher ($*$)	Larger ($*$)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=27.194, c=20 (IV)	Lower ($*$)	Larger ($*$) (Most Cases)
η=0.36, λ_{12}^{Q}=0.0089, λ_{21}^{Q}=0.8402, σ_1=0.73485, σ_2=0.83066, μ_1=47.194, c=10 (V)	Higher ($*$)	Smaller ($*$)

Note: $\star$ and $*$ mean comparing to the benchmark for the base regime and spike regime respectively. Other benchmark parameter values are given in Table 3 and the corresponding results are reported in Table 4.

TABLE 12: *Base Regime as Single Regime for the Regime Switching Model*

Case		No Ramping Restrictions	5000 (CFS-hr)	3000 (CFS-hr)	1000 (CFS-hr)	250 (CFS-hr)
Value and Change of Value at Time 0 When the Initial Price is 40 EUR/MWh						
HF	Value	1329400	1325700	1318800	1307700	1282800
	%ch	N/A	-0.3	-0.8	-1.6	-3.5
FF	Value	1328100	1323300	1314200	1286700	1228700
	%ch	N/A	-0.4	-1.1	-3.1	-7.5
Value and Change of Value at Time 0 When the Initial Price is 80 EUR/MWh						
HF	Value	1363900	1359500	1351700	1338900	1315300
	%ch	N/A	-0.3	-0.9	-1.8	-3.6
FF	Value	1366100	1361900	1354300	1334700	1284400
	%ch	N/A	-0.3	-0.9	-2.3	-6.0

Note: HF means half release rate and full reservoir level; FF means full release rate and full reservoir level. Value is in Euros and %ch refers to the percent change in value compared to the case of no ramping restrictions.