Separation Without Exclusion in Financial Insurance

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Abstract

This paper develops a model of linearly priced financial insurance sold by default-prone insurers. It shows that when insurers differ in their default probabilities there can exist equilibria in which different risk types partially or completely self-sort into insurance contracts offered by different insurers. Partial separation can occur when insurer default and insurance risks are uncorrelated. Full separation is possible when they are correlated. For example, low-risk insured parties may match with higher default-risk insurers, while high-risk insured parties match with lower default-risk insurers.

Keywords: Insurance, Separation, Exclusion, Counterparty Risk, Insured-Insurer Matching.

JEL Classification Numbers: G22, D82, G21.

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1 Introduction

In their seminal papers, Rothschild and Stiglitz (1976) and Wilson (1977) develop a framework in which insurers can screen buyers through a menu of contracts wherein low-risk individuals purchase relatively cheaper insurance than high-risk individuals in return for less coverage. Existence of this type of separating equilibrium depends, crucially, on the assumption that contracts are exclusive. Without exclusivity, prices are typically assumed to be linear, and separation of risk types through a market mechanism cannot generally be achieved.

We model a market for financial insurance that contains two non-standard assumptions: insurance providers may default on their obligations, and pricing of contracts is linear. We show that privately-known type information can be revealed in equilibrium, despite linear pricing. Our insured party (buyer) can be 'risky' or 'safe', with the former more likely to experience a loss. The buyer chooses how much to insure with a stable 'good' insurer versus how much to insure with an unstable 'bad' insurer, where the latter is more likely to default and not pay out claims. Similar to the more standard environment described above, in which buyers are screened using menus of contracts, in our model buyers may "sort" based on insurer choice. For example, risky buyers may insure exclusively with good insurers for a higher premium but reduced likelihood of contract nonperformance (i.e., more coverage). Safe buyers, on the other hand, may contract exclusively with bad insurers for a lower premium, while accepting a higher risk of insurer default (i.e., less coverage).

The possibility of insurer default and restriction to linear pricing are assumptions that are well motivated empirically. First, insurer default, or counterparty risk, is an important feature of financial insurance markets, as became clear in the wake of the recent credit crisis, in which numerous sellers of protection simultaneously experienced extreme instability.[1] Regarding the market for credit default swaps (CDS), for example, Arora et al. (2009) and Morkoetter et al. (2012) report that counterparty risk is priced, meaning that less stable insurers charge lower prices (premia) to compete in the market. Second, in markets for financial insurance (as in life insurance),

[1]For example, AIG and many monoline insurers experienced a crisis when too many written contracts required payment. To focus ideas in this paper, we equate counterparty risk with the event of insurer default, but our results are actually more general. For example, even though the monolines did not formally default, their mere downgrade required many parties to replace their contracts with a higher-quality insurer. As the number of high-quality insurers became scarce while demand for them increased, many buyers no doubt suffered a cost when replacing their contracts. The insurance literature also offers other sources of risk which could substitute for counterparty risk in our model. For example, Bourgeon and Picard (2012) analyze a case in which insurers can decrease the value of a contract by 'nitpicking'. In the context of financial insurance, presumably some insurers are more likely than others to contest whether a default event has occurred, which can be particularly troublesome in a bilateral contract written on a non-standard risk.

sellers do not restrict buyers from purchasing insurance elsewhere, either because such provisions are not enforceable or are not practical. Thus, non-linear pricing is not a realistic assumption.

We show that there are two types of pooling equilibria that may exist in our model: one in which both buyer types purchase exclusively bad insurance, and another in which they both purchase exclusively good insurance. When the bad insurer can offer a price that is significantly lower than that of the good insurer, with a risk of default that is not too high, then a pooling equilibrium at the bad insurer can exist. On the other hand, if the bad insurer is able to offer little price advantage (or if the price offered by the good insurer is lower), then both buyer types will choose to contract solely with the good insurer, and so a pooling price prevails.

We also show that another type of equilibrium, a semi-separating equilibrium, can exist. In this case, the risky buyer purchases insurance from both the good and bad insurer, while the safe buyer purchases insurance only from the bad insurer. The risky buyer pays more for good insurance than for bad, since they have a higher willingness to pay for good insurance, but also always purchases at least some bad insurance because of the cheaper price. Meanwhile, the safe buyer accepts increased counterparty risk at the bad insurer in exchange for a cheaper premium. A fully separating equilibrium does *not* exist when the buyers loss and insurers default risk are uncorrelated.

The results described above are derived in the case where buyer losses are uncorrelated with insurer default risk. When buyer losses can be correlated with the risk of insurer default, and correlations may depend on the type of buyer, we identify two types of fully-separating equilibria that may exist. When the bad insurer is *more* correlated with the risky buyer than the safe buyer, we demonstrate that a separating equilibrium can exist in which the safe buyer purchases exclusively bad insurance, while the risky buyer purchases only good insurance. In this case, buyer type is revealed to the insurers and prices are determined accordingly. When the bad insurer is *less* correlated with the risky buyer than with the safe buyer, we demonstrate that a separating equilibrium can exist in which the safe buyer insures solely with the good insurer, while the risky buyer purchases only bad insurance. These two fully-separating equilibria can, more generally, be thought of as a *matching* outcome. Even with linear pricing, insured parties can have idiosyncratic reasons to choose one insurer over another.

Allowing for correlation between the default risk and the loss being insured is well supported empirically. The literature has overwhelmingly stressed that the correlation between buyer and insurer is vital to evaluating the counterparty risk to which the buyer is exposed. Examples include

Hull and White (2001), which models default correlations, and Gregory (2011), which reports (pg. 331) "Buying CDS protection represents a very definite form of wrong-way risk that is made worse as the correlation between the credit quality of the reference entity [buyer] and the counterparty increases".

We conclude the analysis by discussing the role of mutualization of counterparty risk, which can occur as a consequence of explicit or implicit bailouts (as in the case of AIG, see Harrington (2009)) or can result from the formation of a central counterparty. Mutualization of counterparty risk distorts the optimal choice of insurance by making counterparty risk homogeneous. In particular, the added value of contracting with a good rather than a bad insurer (reduced counterparty risk) vanishes; consequently, the bad insurer can dominate the market.

As discussed above, our analysis departs from the majority of the insurance literature in two important ways: we allow for insurer default, and we consider contracts that are non-exclusive and priced linearly.

There is a small literature that studies the implications of insurer default for insurance market equilibria. These include Doherty and Schlesinger (1990), Agarwal and Ligon (1998), Phillips, Cummins and Allen (1998), Biffis and Millossovich (2011), and Stephens and Thompson (2011). Most notably, Smith and Stutzer (1990) and Ligon and Thistle (2005) study the implications of default risk for "separation" of types. None of these papers, however, studies the implications in the context of non-exclusivity as we do here.

Almost all papers with non-exclusive contracting assume (explicitly or tacitly) that this feature precludes separation of types (see, for example, Hoy and Polborn (2000), Villeneuve (2003), and Polborn et al. (2006)). An important recent exception is Rothschild (2013), which studies an insurance market with linear pricing in which buyers have multiple risks over which they desire insurance, and where risks are heterogeneous across contingencies. As in our paper, Rothschild (2013) demonstrates that it is differences in coverage over different "types of risk" that achieve separation. The key distinction in our paper is that the coverage differences arise because of intrinsic heterogeneity across insurers, each of which offers a single contract. In Rothschild (2013), the coverage differences are actively selected by insurers for the explicit purpose of separation (i.e., for "screening").

The paper proceeds as follows. Section 2 introduces the basic model framework, and section 3 characterizes various equilibria when insurer default risk is uncorrelated with the risk being insured. Section 4 considers the more general case in which buyer risk and insurer default risk may be correlated, while section 5 provides a discussion on the mutualization of counterparty risk. Concluding remarks are made in section 6, and non-trivial proofs can be found in the appendix section 7.

2 The Model

The basic model is a generalization of that used in Stephens and Thompson (2011). There are two types of agents: insurers (sellers) and a buyer (insured party). This section describes the buyer and insurer problem as well as the equilibrium concept.

2.1 Insurers

There are two risk-neutral insurers, denoted (G)ood and (B)ad, where insurer type is indexed $j \in \{G, B\}$. To introduce counterparty risk, we let insurers default with probability $1 - q_j$, in which case they receive a payoff of zero and default on any claims.[2] Let $q_G = 1$ and $q_B < 1$, so that insurance sold by the good insurer is perfectly safe, while insurance sold by the bad insurer is not. We assume that insurer type is public information.[3] When entering into a contract with a buyer, each insurer forms a belief b_j as to the probability of a claim conditional on that insurer not defaulting. Thus, $1 - b_j$ represents the probability of no claim conditional on the insurer not defaulting. Insurer j charges $P_j < 1$ per unit of protection in return for a contingent payment γ_j, thus expected profit from a contract is

$$\Pi_j = q_j \left[(1 - b_j)(1 + r_j)P_j\gamma_j + b_j((1 + r_j)P_j\gamma_j - \gamma_j) \right], \tag{1}$$

[2] We assume that the contracts considered here have no bearing on the probability of insurer default, which substantially simplifies the analysis and is reasonable when the contract is a relatively small proportion of the insurer's portfolio. To explore the implications of relaxing this we refer the reader to Stephens and Thompson (2011), which allows for this feature. We also assume for simplicity that there is no collateral and no recovery value when insurers default. Allowing for partial recovery and collateral has no interesting qualitative effects. Importantly, if collateral is inserted into the model, it cannot be perfect, thus mitigating all counterparty risk. Since collateral tends to decrease in value in times of stress, assuming that collateral can always perfectly eliminate counterparty risk is not realistic.

[3] Although we ignore the potential asymmetric information regarding insurer type in this paper, the interested reader can see Stephens and Thompson (2011) for a treatment of this issue. The goal of our paper is to show how buyer type can be revealed by market forces. Insurer type cannot be revealed in such a way; however, a cost to buyers of discovering the insurer type could be included in the model to allow buyers to learn the insurer type. Furthermore, it seems plausible that the asymmetric information of the asset risk could be the more important information problem of the two. For example, if one considers insurance being written on complicated financial products (such as CDOs), in all likelihood the asymmetric information problem on the CDO will be more severe than any potential asymmetric information problem of the insurer. This is because the insurer would likely have better ratings coverage and may even have an active CDS market (i.e., contracts actively traded on the risk of the insurer itself) to better ascertain their credit quality. On the other hand, the available information on the CDO would likely be substantially less.

where $r_j > -1$ captures an exogenous unit cost or return from providing the contract. Positive values of r_j can be interpreted as return on investment. Negative values of r_j can be interpreted as, for example, administrative or risk management costs.[4] The first term in expression (1) represents the state in which no claim is made (and the insurer is solvent) wherein the insurer simply receives the premium $P_j\gamma_j$. The second term captures the state in which a claim is made (and the insurer is solvent) wherein the insurer must make the payment γ_j.

We assume that there are potentially many identical insurers of either type, so that the market for insurance is competitive and the expected return to a contract is zero from the insurers' perspective.[5]

2.2 Buyer

There is a single buyer who is endowed with an asset that can be one of two types with equal probability: (S)afe or (R)isky, denoted $i \in \{S, R\}$. Asset i provides positive return w with probability p_i and zero with probability $1 - p_i$, where $1 > p_S > p_R > 0$. Henceforth, we refer to a buyer with a safe type of asset as safe, and a buyer with a risky type of asset as risky. We assume that buyer type is private information.

The buyer may purchase insurance against the zero payoff state from either insurer, where we denote coverage for type i at insurer j by γ_j^i. Define q_B^i as the probability that the bad insurer is solvent conditional on a claim from buyer i. Thus, $1 - q_B^i$ represents the probability that the bad insurer defaults conditional on a claim from buyer i. As discussed above, we assume throughout the paper that the good insurer is completely safe, so that $q_G = q_G^R = q_G^S = 1$. A buyer of type i has expected utility (2), where u is increasing and strictly concave:

$$\begin{aligned} V^i = p_i u(w - P_B\gamma_B^i - P_G\gamma_G^i) + (1-p_i)[q_B^i u((1-P_G)\gamma_G^i + (1-P_B)\gamma_B^i) \\ +(1-q_B^i)u((1-P_G)\gamma_G^i - P_B\gamma_B^i)]. \end{aligned} \tag{2}$$

The first term in V^i represents the state of the world in which no claim is made. The payoff in this state is w less the insurance premium paid to both types of insurers, $P_B\gamma_B^i + P_G\gamma_G^i$. The second

[4]The most interesting case is presumably that in which $r_B > r_G$, which implies that the bad insurer's zero-profit price can be lower than that of the good insurer. Thus, the title of *bad* insurer refers only to the higher counterparty risk of that insurer type. As we will show below, the lower premium that bad insurers may charge can attract the buyer.

[5]Although we simplify the analysis and focus on only two insurers, one can think of each as a large number of identical insurers, which more clearly rules out a non-linear pricing scheme. This approach was taken in an earlier version of the paper, but had no interesting qualitative implications.

term in V^i captures the state in which a claim is made. In this case, with probability q_B^i, the buyer's claims are both paid, whereas with probability $1 - q_B^i$, only the good insurer pays the claim (assuming protection is purchased from both insurers).

2.3 Equilibrium

An equilibrium is a set of contracts for buyer type i denoted $\gamma^i = [\gamma_B^i, \gamma_G^i]$, a set of prices $P = [P_B, P_G]$, and beliefs $b = [b_B, b_G]$, such that:

i. Buyer type i chooses $\gamma^i \geq 0$ to maximize V^i, given P.

ii. Insurers earn zero profits given their beliefs, i.e., $\Pi_G = \Pi_B = 0$.

iii. Beliefs are consistent: If $\gamma_j^R + \gamma_j^S > 0$ then $b_j = \frac{(1-p_R)(q_j^R/q_j)\gamma_j^R + (1-p_S)(q_j^S/q_j)\gamma_j^S}{\gamma_j^R + \gamma_j^S}$.

Note that $(1 - p_i)(q_j^i/q_j)$ represents the probability that buyer type i submits a claim, conditional on insurer j being solvent. Using expression (1), the zero profit premium is $b_j/(1 + r_j)$.

3 No Correlation

In this section we consider the case in which there is no correlation between the risk being insured and the risk of default at the bad insurer, therefore $q_B^R = q_B^S = q_B$.

3.1 Pooling

Consider the case in which both buyer types contract solely with the good insurer, so that $\gamma_G^S > 0$, $\gamma_G^R > 0$, $\gamma_B^S = 0$, and $\gamma_B^R = 0$. Alternatively, both buyer types can insure solely with the bad insurer, in which case $\gamma_G^S = 0$, $\gamma_G^R = 0$, $\gamma_B^S > 0$, and $\gamma_B^R > 0$. The following proposition shows that either of these two types of pooling equilibria may exist.

Proposition 1

i. There exists a $u()$, $q_B = q_B^R = q_B^S$, p_S, p_R, r_G, r_B, and w for which there is a pooling equilibrium in which both buyer types purchase a positive amount of insurance from the good insurer and nothing from the bad insurer.

ii. There exists a $u()$, $q_B = q_B^R = q_B^S$, p_S, p_R, r_G, r_B, and w for which there is a pooling equilibrium in which both buyer types purchase a positive amount of insurance from the bad insurer and nothing from the good insurer.

Proof. *See Appendix.*

To prove this proposition we construct two examples. These two pooling equilibria are fairly intuitive. If, for example, r_G and r_B are such that $P_G < P_B$, then it is obvious that no buyer would contract with the bad insurer since that insurer provides lower quality protection and charges a higher price. In this case, a pooling equilibrium at the good insurer may exist. On the other hand, as $q_B \to 1$, and r_B and r_G are such that $P_B < P_G$, then no buyer would contract with the good insurer since both insurers provide essentially the same quality of protection, while the bad insurer charges a lower price. Consequently, a pooling equilibrium at the bad insurer can exist in this case.

3.2 Semi-Separation

We show that there can exist a semi-separating equilibrium in which only the risky buyer purchases a positive amount of insurance with the good insurer, while both the risky and safe buyers purchase a positive amount of insurance with the bad insurer. In this case $\gamma_G^S = 0$, $\gamma_G^R > 0$, $\gamma_B^S > 0$, and $\gamma_B^R > 0$. Thus, the good insurer knows that its buyer is risky, while the bad insurer cannot distinguish between buyer types.[6]

Proposition 2 *There exists a $u()$, $q_B = q_B^R = q_B^S$, p_S, p_R, r_G, r_B, and w for which there is a semi-separating equilibrium in which both buyer types purchase a positive amount of insurance from the bad insurer, while only the risky buyer purchases a positive amount of insurance from the good insurer.*

Proof. *See Appendix.*

To prove this proposition we construct an example. This proposition establishes that for certain parameters at least one semi-separating equilibrium can exist. Intuitively, in this equilibrium, bad insurance is sufficiently cheap that it is purchased by both buyer types. However, counterparty risk is more costly for the risky buyer, since this party is more likely to make a claim. Thus, the risky buyer is willing to purchase some insurance from the good insurer, even though the good insurer will infer that any buyer purchasing good insurance is the risky type and, consequently, will price

[6]It is important to reiterate that for this to be true, we assume that the bad insurer cannot infer buyer type from the size of the contract, which will generally differ across types. As discussed, considering multiple insurers of each type can justify the linear pricing assumption.

the contract accordingly. On the other hand, the price at the good insurer is too high for the safe buyer and so this buyer purchases coverage only from the bad insurer.

A natural question is whether full separation of types can occur. A candidate full separating equilibrium is one in which the risky buyer purchases only good insurance, while the safe buyer purchases only bad. In this case, consistency of beliefs requires that $b = [1 - p_S, 1 - p_R]$. Since the bad insurer offers a riskier product, the bad insurer must have a higher expected return if the safe buyer is to purchase a positive amount of insurance in equilibrium. On the other hand, the risky buyer is fully insured at the good insurer and thus is locally risk neutral. Buying a small amount of higher return insurance from the bad insurer would therefore increase that buyer's utility. The case in which the risky buyer contracts with the bad insurer and the safe buyer with the good insurer is similar. The following proposition summarizes this intuition.

Proposition 3 *A full separating equilibrium in which either (i) the risky buyer purchases a positive amount of insurance solely from the good insurer and the safe buyer purchases a positive amount of insurance solely from the bad insurer, or (ii) the risky buyer purchases a positive amount of insurance solely from the bad insurer and the safe buyer purchases a positive amount of insurance solely from the good insurer, does not exist.*

The independence between buyer and insurer risk is a key reason that a full separating equilibrium does not exist. However, the independence assumption is unlikely to hold since counterparty risk is often correlated with the underlying asset. For example, when both buyer and insurer are exposed to market or aggregate risk. In the more general model specification which includes this feature, the value of bad insurance for each buyer type can differ because of different correlations between the risk being insured and the counterparty risk of the insurer.

4 Correlation and Full Separation

We now analyze the more general case in which the asset of the buyer can be correlated to the default risk of the bad insurer. This leads us to a key result of the paper, namely that differences in counterparty risk allow for the possibility of a full separating equilibrium in which the insurers learn the true probability of a claim. First consider potential differences in default risks captured by q_B^R and q_B^S. Empirical evidence suggests that risky firms tend to be highly correlated (see for example, Campbell et al. (2008)), so that $q_B^R < q_B^S$ represents the most plausible case, i.e., the risky buyer is more correlated with the bad insurer than is the safe buyer. Using an example in which $q_B^R < q_B^S$, we show that a separating equilibrium in which the risky buyer insures with only the good insurer, while the safe buyer insures with only the bad insurer can exist. Alternatively, using an example in which $q_B^R > q_B^S$, we show that a separating equilibrium in which the risky buyer insures with only the bad insurer, while the safe buyer insures with only the good can exist. The following proposition formalizes this result.

Proposition 4

i. There exists a $u()$, q_B, q_B^S, q_B^R, p_S, p_R, r_G, r_B, and w for which there is a separating equilibrium in which the safe buyer purchases a positive amount of insurance solely from the bad insurer and the risky buyer purchases a positive amount of insurance solely from the good insurer.

ii. There exists a $u()$, q_B, q_B^S, q_B^R, p_S, p_R, r_G, r_B, and w for which there is a separating equilibrium in which the risky buyer purchases a positive amount of insurance solely from the bad insurer and the safe buyer purchases a positive amount of insurance solely from the good insurer.

Proof. *See Appendix.*

A full separating equilibrium can exist because the value of protection at each insurer is different for each buyer type. One can think of this as the buyer *matching* with the insurer with whom it is the least correlated. In other words, when there are idiosyncratic reasons for buyers to prefer a certain insurer type, the traditional insurance framework is significantly altered so as to permit a full separating equilibrium, even with linear pricing.

5 Bailouts and Central Counterparties

Since the onset of the credit crisis, the debate over implicit/explicit government bailouts has been front and center. In hopes of reducing the need for government bailouts, and under the broader goal of financial stability, the U.S. is moving some financial contracts to central counterparties. It is straightforward to discuss government bailouts and central counterparties given our previous analysis, since they both serve to mutualize counterparty risk.[7] When the government can step

[7] In related work, Rymaszewski et al. (2012) study insurance guaranty funds as a mechanism of mutualizing risk.

in to prevent failure of the insurer, the mechanism of mutualization is obvious: the taxpayers will rescue a defaulted insurer so that buyers of protection do not suffer counterparty risk. A Central Counterparty (CCP) also serves to mutualize counterparty risk, except in this case the risk is shared by all participants, rather than by taxpayers as in the previous scenario.[8]

Consider the effect that mutualization may have in our environment. Although the counterparty risk is mutualized, the contract is still written between two parties; thus, the price can still vary based on the insurer. Now consider the counterparty risk to which the buyer is exposed. In the case of full mutualization, the risk to which a buyer is exposed is no longer that of the individual insurer. Therefore, when choosing an insurer, buyers will not consider the individual counterparty risk of each insurer, but will respond only to the price offered.[9] In this case, bad insurers able to offer a cheaper price will dominate the market and drive out the good insurers in what amounts to a simple application of Gresham's Law.

6 Conclusion

We analyze a simple market for financial insurance with linear pricing when insurers can default. We show that, in contrast with canonical insurance models, a separating equilibrium can exist in which all insured parties reveal private type information based on the type of insurer with which they contract. To obtain this result, we require that the risk being insured and the insurer default risk is correlated. When these risks are uncorrelated, we show that a semi-separating equilibrium can exist.

Going forward, there are other possible applications of the results of this paper. For example, life annuities are a natural application since (i) they are non-exclusive and linearly priced (see, for example, Hoy and Polborn (2000) and Villeneuve (2003)), and (ii) they are extremely long-term contracts, so default risk is a potentially serious problem.

[8] For a general overview of CCPs, see Bliss and Steigerwald (2006).

[9] In the case of a CCP, assume that each contract has a negligible effect on the counterparty risk of the CCP.

7 Appendix

Proof of Proposition 1

Define c_ω^i to be the consumption for buyer type i in state $\omega \in \{0,1,2\}$, where $\omega = 0$ is the no claim state, $\omega = 1$ is the state in which a claim is made and the bad insurer is solvent, and $\omega = 2$ represents the state in which a claim is made and the bad insurer defaults. Equations (3) and (4) characterize the partial derivatives of V^i, when the utility function is $u(c_\omega^i) = log(1 + c_\omega^i)$.

$$V_{\gamma_G}^i = -\frac{p_i P_G}{1+c_0^i} + \frac{(1-p_i)q_B^i(1-P_G)}{1+c_1^i} + \frac{(1-p_i)(1-q_B^i)(1-P_G)}{1+c_2^i},\ i \in \{R,S\} \tag{3}$$

$$V_{\gamma_B}^i = -\frac{p_i P_B}{1+c_0^i} + \frac{(1-p_i)q_B^i(1-P_B)}{1+c_1^i} - \frac{(1-p_i)(1-q_B^i)P_B}{1+c_2^i},\ i \in \{R,S\} \tag{4}$$

We prove by example. First consider the good insurance pooling equilibrium. Let $u(c_\omega^i) = log(1 + c_\omega^i)$, $p_S = 0.75$, $p_R = 0.65$, $q_B = q_B^R = q_B^S = 0.7$, $q_G = 1$, $r_B = r_G = 0$, and $w = 1$. We assert that $P_G \approx 0.3233$, $P_B = 0.25$, $\gamma_G^R \approx 1.2044$, $\gamma_G^S \approx 0.4380$, $\gamma_B^R = 0$, $\gamma_B^S = 0$, $b_B = 1 - p_S = 0.25$, and $b_G = (1-p_S)\gamma_G^S/(\gamma_G^S + \gamma_G^R) + (1-p_R)\gamma_G^R/(\gamma_G^S + \gamma_G^R) \approx 0.3233$ is an equilibrium since (1) This satisfies $V_{\gamma_G}^S = 0$, $V_{\gamma_G}^R = 0$, $V_{\gamma_B}^S \approx -0.0141 < 0$, and $V_{\gamma_B}^R \approx -0.0141 < 0$, which are necessary and sufficient for individual optimization, (2) Insurers earn zero profits given b_G and b_B, and (3) b_G is consistent by construction while b_B is trivially consistent since $\gamma_B^S + \gamma_B^R = 0$.

Now consider the bad insurance pooling equilibrium. Let $u(c_\omega^i) = log(1 + c_\omega^i)$, $p_S = 0.75$, $p_R = 0.65$, $q_B = q_B^R = q_B^S = 0.99$, $q_G = 1$, $r_B = 0.5$, $r_G = 0$, and $w = 1$. We assert that $P_G = 0.25$, $P_B \approx 0.2091$, $\gamma_B^R \approx 2.4611$, $\gamma_B^S \approx 1.4065$, $\gamma_G^R = 0$, $\gamma_G^S = 0$, $b_G = 1 - p_S = 0.25$, and $b_B = (1-p_S)\gamma_B^S/(\gamma_B^S + \gamma_B^R) + (1-p_R)\gamma_B^R/(\gamma_B^S + \gamma_B^R) \approx 0.3136$ is an equilibrium since (1) This satisfies $V_{\gamma_G}^S \approx -0.0194 < 0$, $V_{\gamma_G}^R \approx -0.0158 < 0$, $V_{\gamma_B}^S = 0$, and $V_{\gamma_B}^R = 0$, which are necessary and sufficient for individual optimization, (2) Insurers earn zero profits given b_G and b_B, and (3) b_B is consistent by construction while b_G is trivially consistent since $\gamma_G^S + \gamma_G^R = 0$.

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Proof of Proposition 2

We prove by example. Let $u(c_\omega^i) = log(1 + c_\omega^i)$, $p_S = 0.75$, $p_R = 0.65$, $q_B = q_B^R = q_B^S = 0.9$, $q_G = 1$, $r_B = 0.2$, $r_G = 0$, and $w = 1$. We assert that $P_G = 0.35$, $P_B \approx 0.2655$, $\gamma_G^R \approx 0.0337$, $\gamma_G^S = 0$, $\gamma_B^R \approx 1.2737$, $\gamma_B^S \approx 0.5821$, $b_G = 1 - p_R = 0.35$, and $b_B = (1-p_S)\gamma_B^S/(\gamma_B^S + \gamma_B^R) + (1 - p_R)\gamma_B^R(\gamma_B^S + \gamma_B^R) \approx 0.3186$ is an equilibrium since (1) This satisfies $V_{\gamma_G}^S \approx -0.0206 < 0$, $V_{\gamma_G}^R = 0$,

$V^S_{\gamma_B} = 0$, and $V^R_{\gamma_B} = 0$, which are necessary and sufficient for individual optimization, (2) Insurers earn zero profits given b_G and b_B, and (3) b_G and b_B are consistent by construction.

$\blacksquare$

Proof of Proposition 4

We prove by example. Consider the first case. Let $u(c^i_\omega) = log(1 + c^i_\omega)$, $p_S = 0.75$, $p_R = 0.65$, $q_G = 1$, $q^S_B = q_B = 0.8$, $q^R_B = 0.7$, $r_B = r_G = 0$, and $w = 1$. We assert that $P_G = 0.35$, $P_B = 0.25$, $\gamma^R_G = 1$, $\gamma^S_G = 0$, $\gamma^R_B = 0$, $\gamma^S_B \approx 0.4342$, $b_B = 1-p_S = 0.25$, and $b_G = 1-p_R = 0.35$ is an equilibrium since (1) This satisfies $V^S_{\gamma_G} \approx -0.0043 < 0$, $V^R_{\gamma_G} = 0$, $V^S_{\gamma_B} = 0$, and $V^R_{\gamma_B} \approx -0.0030 < 0$, which are necessary and sufficient for individual optimization, (2) Insurers earn zero profits given b_G and b_B, and (3) b_G and b_B are consistent by construction.

Now consider the second case. Let $u(c^i_\omega) = log(1+c^i_\omega)$, $p_S = 0.75$, $p_R = 0.65$, $q_G = 1$, $q^S_B = 0.65$, $q^R_B = q_B = 0.9$, $r_B = 0.45$, $r_G = -0.3$, and $w = 1$. We assert that $P_G \approx 0.3571$, $P_B \approx 0.2414$, $\gamma^R_G = 0$, $\gamma^S_G \approx 0.2333$, $\gamma^R_B \approx 1.5470$, $\gamma^S_B = 0$, $b_G = 1 - p_S = 0.25$, and $b_B = 1 - p_R = 0.35$ is an equilibrium since (1) This satisfies $V^S_{\gamma_G} = 0$, $V^R_{\gamma_G} \approx -0.0136$, $V^S_{\gamma_B} \approx -0.0056$, and $V^R_{\gamma_B} = 0$, which are necessary and sufficient for individual optimization, (2) Insurers earn zero profits given b_G and b_B, and (3) b_G and b_B are consistent by construction.

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