

COUNTERPARTY RISK IN FINANCIAL CONTRACTS: SHOULD THE INSURED WORRY ABOUT THE INSURER?*

JAMES R. THOMPSON

We analyze the effect of counterparty risk on financial insurance contracts, using the case of credit risk transfer in banking. This paper posits a new moral hazard problem on the insurer side of the market, which causes the insured party to be exposed to excessive counterparty risk. We find that this counterparty risk can create an incentive for the insured party to reveal superior information about the likelihood of a claim. In particular, a unique separating equilibrium may exist, even in the absence of any costly signalling device.

I. INTRODUCTION

In May 2007 an agreement for an insurance contract was reached between UBS, a large multinational bank and Paramax Capital, a group of hedge funds. The notional amount of protection (against risk related to subprime mortgages) that Paramax provided for UBS was \$1.31 billion. What was startling was that Paramax had only \$200 million of capital to insure the risk. When claims were submitted by UBS soon after the contract was signed, Paramax was unable to fulfil them.¹ In another case from late 2007, it was revealed that ACA financial guaranty had sold protection totalling \$59 billion while possessing capital resources of only \$425 million (Das (2008)). The issue of counterparty risk has been given considerable media attention due to the financial crisis of 2007-2009. Many firms who issue insurance contracts (e.g., AIG, many monoline insurers like Ambac and MBIA²) have experienced nothing short of a crisis as they try to pay claims to insured parties. Recently, the US government has even considered providing an explicit backstop for

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¹*Financial Times* (FT.com). June 01, 2008. "First Comes the Swap. Then It's the Knives."

²Monoline insurers guarantee the timely repayment of bond principal and interest when an issuer defaults.

AIG to protect against future losses from financial insurance claims.³ Many banks who purchased credit risk insurance have found themselves greatly exposed to potentially unstable insurers. For example, as of June 2008, UBS is estimated to have had \$6.4 billion of risk ceded to monolines while Citigroup and Merrill Lynch had \$4.8 billion and \$3 billion respectively.⁴

In this paper, we develop an agency model to analyze an insurer's optimal investment decision when failure is a possibility. We demonstrate that an insurer's investment choice may be inefficient so that a moral hazard problem exists on this side of the market. This insurer moral hazard problem is shown to have an upside as it can alleviate the adverse selection problem on the part of the insured. In constructing the model, we will not attempt to incorporate all the salient features of the various insurance players and markets. Instead, we will focus on a general insurance problem using the market for credit risk transfer as our example. In taking a general approach, one of the novelties of our paper is that we inject the insurers' asset-liability management into a standard model of insurance.

The market for risk protection is one of the most important markets available today. Figure I shows the growth rate in credit derivatives since 2003.⁵ It is easy to see the rapid growth that these financial markets have experienced. An institution on which these markets have a particularly profound effect is the banking system. The reason is that banks were once confined to a simple borrow short and lend long strategy. However, they can now disperse credit risk through credit derivatives markets to better implement risk management policies. This in itself may be a positive development; however, two features make these markets potentially different (and dangerous) when compared to traditional insurance markets. First, the potential for unstable counterparties, since potentially large credit risks can be ceded to parties such as hedge funds which may or may not be in a better position to handle them.⁶ The second feature which is unique to this market is the large size of the contracts.⁷ It would seem prudent then to ask the question of how stable are the insurers and what are their incentives? This entails a study of counterparty risk. In what is to follow, we define counterparty risk as the risk that when a claim is made, the insurer is unable to fulfil its obligations.

This paper posits a new moral hazard problem which leads to a novel result. What is new is that the moral hazard problem occurs on the part of the insurer because it may choose an excessively risky portfolio.⁸ The intuition is as follows. The insurer has a random return and may

³*Bloomberg* (Bloomberg.com). February 26, 2009. "AIG Rescue May Include Credit-Default Swap Backstop."

⁴*Financial Times* (FT.com). June 10, 2008. "Banks face \$10bn monolines charges."

⁵A credit derivative, and specifically a *credit default swap* is an instrument of credit risk transfer whereby an insurer agrees to cover the losses of the insured that take place if pre-defined events happen to an underlying borrower. (In many cases, this event is the default of the underlying bond. However, some contracts include things like restructuring and ratings downgrades as triggering events.) In exchange for this protection, the insured agrees to pay an ongoing premium at fixed intervals for the life of the contract.

⁶The Fitch agency reports that banks are the largest insured party in this market. Banks and hedge funds are the largest insurers, followed by insurance companies and other financial guarantors. See Fitch Ratings: Financial Institutions. 2006. "Global Credit Derivatives Survey: Indices Dominate Growth and Banks' Risk Position Shifts."

⁷The two typical credit default swap contract denominations are \$5 and \$10 million. The total size of a given contract can be much larger as will be discussed below.

⁸The incorporation of the traditional moral hazard problem on the part of the insured is discussed in section V.

fail regardless of whether it sells the insurance contract. By selling insurance, it receives a premium that it invests in liquid or illiquid assets. This investment choice affects the probability it will fail. In particular, what reduces the probability of failure the most in the state in which a claim is not made, makes it more likely that the insurer will fail in the state in which it is. For example, if the insurer believes that the contract is relatively safe, it may be optimal to put capital into less liquid assets to reap higher returns. This then lowers the chance of failure in the state in which a claim is not made. However, assets which yield these higher returns can also be more costly to liquidate, and therefore make it more difficult to free up capital if a claim is made. The moral hazard arises because the premium is not made conditional on an observed outcome, rather it is paid upfront.⁹ Therefore, there is no way for the insured party to influence the insurer's investment decision. We show that the resulting equilibrium is inefficient.

It is important to note that the excess risk taking (moral hazard) is not a direct consequence of a limited liability assumption on the insurer. One of the first discussions concerning the connection between limited liability and excess risk taking can be found in Jensen and Meckling (1976). In that paper, a manager does not have the proper incentives to internalize the downside risk of a firm so that they take excessive gambles. In our paper however, we will show that the inefficiency arises because the insurer does not internalize the counterparty risk that it causes to the insured party. It is this fact that makes the insurer take excess risk. This is not meant to imply however, that limited liability does not play a role in the risk taking behavior of insurers. Rather, it does not play a major role in terms of the inefficiency we analyze.

The key result of the paper comes as a consequence of the insurer's investment strategy described above. Starting from a standard adverse selection problem, with the agent (insured party) having the informational advantage, then there is a potential that the adverse selection problem is ameliorated if the principal (insurer) has a moral hazard problem. In particular, consider the adverse selection problem that may be present because of the superior information that the insured has about the probability of a claim. Akerlof (1970) describes the dangers of informational asymmetries in insurance markets. In his seminal paper, it is shown how the market for good risks may break down, and one is left with insurance only being issued on the most risky of assets, or in Akerlof's terminology, lemons. The incentive that underlies this result is that the insured only wishes to obtain the lowest insurance premium. This incentive will still be present in our model; however, we uncover an opposing incentive.

We show that the safer the underlying claim is perceived to be, the more counterparty risk that the insured party is exposed to. This can give the insured party with a poor quality asset an incentive for truthful revelation. We show that this new effect, which we call the *counterparty risk effect* allows a unique separating equilibrium to be possible. This result is new in that separation can occur in the absence of a costly signalling device. After Akerlof (1970) showed that no separating equilibrium can exist, the literature developed the concept of signaling devices with such famous

⁹To have a conditional premium would require a higher payment from the insured to the insurer when the insurer is able to pay than when it is not. This goes against the nature of an insurance contract: when a claim is made, the insured party does not want to pay the insurer.

examples as education as a job market signal in Spence (1973). These papers allow the high (safe) type agents to separate themselves by performing a task which is “cheaper” for them than for the low (risky) type agents. Our paper can achieve separation by the balance between the insured’s desire for the lowest insurance premium, and the desire to be exposed to the least counterparty risk. One can think of this result as adding to the *cheap talk* literature by showing an insurance problem in which costless communication can bring about separation of types.¹⁰ We call this the *separating equilibrium result*. Whereas the moral hazard obtains regardless of the contract size or number of insured parties, the separating equilibrium result holds if one or both of the following two conditions are met. First, if a contract is sufficiently large relative to the size of the insurer so that its investment decision is significantly affected. Second, if there is aggregate private risk shared among a pool of insured parties. We discuss the starkness of the separating equilibrium result in the robustness section V. There we consider the way in which this result would manifest itself in a richer model, wherein partial information revelation is possible.

Consider the first situation in which the insured risk is sufficiently large relative to the size of the insurer. Hedge Funds, being one of the most active players in issuing financial insurance are a good example of when the insurer may be relatively small.¹¹ Barth et. al. (2008) report that the average hedge fund size is \$150 million. With some forms of insurance (e.g., insurance written on collateralized debt obligations (CDOs)), banks often use credit default swaps (CDSs) to hedge large quantities of risk. For example, a bank may engage in a negative-basis trade in which a AAA-rated tranche of a CDO is purchased, and hedged with a CDS (Gorton 2008). According to Benmelech and Dlugosz (2009), the average CDO size is \$591 million of which the AAA-tranche constitutes on average 71.4%. To protect against this risk, a bank would have to purchase at least \$421 million of CDS protection. Although such a large insurance contract may be split over many counterparties, the example at the beginning of the paper constitutes a situation in which it was not. UBS purchased \$1.31 billion of protection from Paramax who had only \$200 million of capital available. This serves as an example of a very large contract and a relatively small insurer which would fit the setup of the basic model of the paper. The basic model of one contract between an insured party and an insurer is also best suited for developing the intuition behind our results, and so is used in section II.

In section IV, we generalize the model to the case of multiple insured parties, each of which is insignificant to the insurer’s investment decision. We consider the case in which the insured parties

¹⁰Crawford and Sobel (1982) is one of the classic works on cheap talk. In that paper, the authors show that in a setting with a continuum of possible senders (experts), only partition equilibria can be attained when the sender and receiver have differing preferences. Interestingly, Krishna and Morgan (2004) show that more information transmission can occur in the Crawford and Sobel (1982) model if multi-stage (but still costless) communication is allowed. In that paper, the key is that the extra round of communication can be used to create uncertainty as to how the receiver will process the information sent by the sender. This is done for fixed preferences of the sender and receiver. In contrast, we will not use multi-stage communication, instead, counterparty risk in our model will act to potentially align the sender and receivers’s preferences thereby improving information transmission. For a general review of the cheap talk literature, see Farrell and Rabin (1996).

¹¹Note that if a hedge fund, acting as an insurer, hedges all insurance risk by purchasing protection from another party, they would not have a meaningful investment choice in our model. The UBS-Paramax case described earlier is an example in which the hedge fund was likely unable to perfectly hedge the risk so acted as a net seller of protection.

share an informational advantage on a common component of risk. This type of private information is best motivated in the context of the credit crisis.¹² One plausible story is that, prior to the onset of the crisis, banks possessed the knowledge that many instruments were riskier than the risk bearers knew, i.e., a correlated informational advantage. There are two reasons for this: First, many banks were more intimately involved in the creation of the risk than various key insurers. Second, many banks were trading similar securities (e.g., tranches of CDOs containing sub-prime mortgages), so that each bank would have knowledge about how their risk is related to other risks that they were ceding, as well as similar risks that other banks were ceding. Gorton (2008) and Jenkinson, Penalver, and Vause (2008) support this systematic informational asymmetry view by analyzing the chain of players that transferred the subprime risk to the eventual end risk taker. They argue that each link in the chain constituted more information loss because parties, such as dealer banks, understood the complexity of the instruments better than the parties with whom the risk was ceded. Furthermore, Ashcraft and Schuermann (2008) report that banks were very active as originators and arrangers of subprime mortgage debt and so were able to gain an informational advantage on the risk. In referring to the parties who sold insurance contracts on this type of risk, Jenkinson, Penalver, and Vause (2008) conclude: *“In practice, though, those most willing to take these risks have in some cases turned out to be those who understood the risks the least...”* This is the kind of setting that the generalization of the model in section IV is designed to analyze.

I.A. RELATED LITERATURE

This paper contributes to two streams of literature: that of credit risk transfer and credit derivatives and that of insurance economics. The literature on credit risk transfer (CRT) is relatively small but is growing. Allen and Gale (2006) motivate a role for CRT in the banking environment while Parlour and Plantin (2008) derive conditions under which liquid CRT markets can exist. Using the same framework as Allen and Gale (2006), Allen and Carletti (2006) show how a default by an insurance company can cascade into the banking sector causing a contagion effect when the two parties are linked through CRT. Wagner and Marsh (2006) argue that setting regulatory standards that reflect the different social costs of instability in the banking and insurance sector would be welfare improving. Our paper differs from the ones above because they do not consider the agency problems of insurance contracts. As a result, they do not discuss the consequences that instability can have on the contracting environment, and how this affects the behavior of the parties involved. Duffee and Zhou (2001), Thompson (2007) and Parlour and Winton (2008) analyze informational problems in insurance contracts; however, they focus on the factors that affect the choice between sales and insurance of credit risk and do not analyze counterparty risk.

We contribute to the literature on insurance economics by raising the issue of counterparty risk which has received little attention. Henriot and Michel-Kerjan (2006) recognize that insurance contracts need not fit the traditional setup in which the insurer is the principal and the insured, the agent. The authors relax this assumption and allow the roles to change. Their paper however,

¹²For a review on the causes and symptoms of the credit crisis see Greenlaw et al. (2008) and Rajan (2008).

does not consider the possibility of counterparty risk. Plantin and Rochet (2007) study and give recommendations on the prudential regulation of insurance companies. Their work does not consider the insurance contract itself under counterparty risk as is done in our paper. Consequently, the authors do not analyze the effects of counterparty risk on the informational problems. Instead, they conjecture an agency problem arising from a corporate governance standpoint. We analyze an agency problem driven entirely by the investment incentives of the insurer. Phillips, Cummings, and Allen (1998) and Myers and Read (2001) study the capital allocation and pricing consequences of having multiple lines of insurance when there is default risk. They do not consider the investment decision nor an agency problem as is analyzed in this paper.

The paper proceeds as follows: Section II outlines the model and solves the insurer's problem. Section III determines the equilibria that can be sustained when asymmetric information is present. We also show a moral hazard problem on the part of the insurer by determining that an inefficient investment choice is made. Furthermore, we perform two welfare analyses. First, we compare the welfare benefit of the separating equilibrium versus the loss due to the moral hazard. Second, we compare the separation mechanism of this paper to the traditional separation mechanism with no moral hazard but costly signals available. Section IV analyzes the case of multiple insured parties. Section V explores the robustness of the model and section VI concludes. Longer proofs are relegated to Appendix A in section VII.

II. THE MODEL SETUP

The model is in three dates indexed $t = 0, 1, 2$. There are two main agent types: multiple insured parties, whom we will call banks, and multiple risk insurers, whom we will call Insuring Financial Institutions (IFIs). For expositional purposes, we will first focus on the case where there is only one bank. In section IV we will allow multiple banks.¹³ We also assume there is an underlying borrower who has a loan with the bank. We model this party simply as a return structure. The size of the loan is normalized to 1. We motivate the need for insurance through an exogenous parameter (to be explained below) which makes the bank averse to risk. We assume there is no discounting; however, adding this feature will not affect our qualitative results.

II.A. THE BANK

The bank is characterized by the need to shed credit (loan) risk. If the bank has a loss for which it is insured, however the insurer cannot pay, it suffers the cost Z . For example, Z could be a regulatory penalty for exceeding some risk level, or an endogenous reaction to a shock to the bank's portfolio; however, we will not model this here. If a bank does not insure at all, it also suffers the cost Z . It is this cost that makes the bank averse to holding the risk and so sheds it

¹³With a minor modification to the setup there is another way to interpret these two cases. In both cases there are multiple banks (or multiple contracts); however, in the first case the risk is perfectly correlated while in the second it is not.

through insurance. There are two types of loans that a bank can insure, a safe type (S) and a risky type (R). A bank is endowed with one or the other with equal probability. The return on either loan is $R_B > 1$ if it succeeds which happens with probability p_S (p_R) if it is safe (risky), where $1 > p_S > p_R > 0$. We assume that the return of a failed loan is zero. The loan type is private knowledge to the bank and reflects the unique relationship between them and the underlying borrower. We assume that the loan can be costlessly monitored, so that there is no moral hazard problem in the bank-borrower relationship. This assumption can be relaxed and is discussed further in section V. Note that there is nothing in the analysis to follow that requires this to be a single loan. When we interpret this as a single loan, the insurance contracts to be introduced in section II.C will resemble that of a *credit default swap*. In the case that this is a return on many loans, the insurance contract will resemble that of a *portfolio default swap* or *basket default swap*.¹⁴

The regulator requires the bank to insure a fixed, and equal proportion of either loan. For simplicity, the bank must insure a proportion γ of its loan, regardless of its type.¹⁵ This assumption shuts down the traditional mechanism for obtaining a separating equilibrium. In standard models of insurance contracts, the traditional separating mechanism involves a costly signaling device such as the amount of insurance that the safe and risky type take on. The safe type is able to signal that it is safe by taking on less insurance (e.g., a higher deductible). Shutting down this mechanism illuminates the separating mechanism that counterparty risk creates. In section III.C we will compare the two mechanisms of separation.

In what follows, we only model the payoff to this loan for the bank; however, it can be viewed as only a portion of its total portfolio. We assume that the bank cannot fail; however, allowing the bank to fail will not affect our qualitative results since it will not affect the insurance contract to be introduced in section II.C. We now turn to the modelling of the IFI.

II.B. THE INSURING FINANCIAL INSTITUTION

We assume that the insurer receives a random return at $t = 2$ governed by the uniform distribution function $F(\theta)$ with upper (lower) bound $\bar{R}_f > 0$ ($\underline{R}_f < 0$). We assume that bankruptcy occurs when the portfolio draw is in the set $[\underline{R}_f, 0]$;¹⁶ however, because of limited liability, the IFI receives

¹⁴A portfolio or basket default swap is a contract written on more than one loan. There are many different configurations of these types of contracts. For example, a first-to-default contract says that a claim can be made as soon as the first loan in the basket defaults.

¹⁵The assumption of a fixed amount of insurance regardless of type is not crucial. We can think of γ being solved for by the bank's own internal risk management. Therefore, we could have a differing γ depending on loan quality. What is important in this case is that the IFI is not able to perfectly infer the probability of loan defaulting from γ . This assumption is justified when the counterparty does not know the exact reason the bank is insuring. To know so would require them to know everything about the bank's operations, which should be excluded as a possibility. In this enriched case, γ can be stochastic for each loan type reflecting different (private) financial situations for the bank. This topic has been addressed in the new Basel II accord which allows banks to use their own internal risk management systems in some cases to calculate needed capital holdings. One reason for this change is because of the superior information banks are thought to have on their own assets; regulators have acknowledged that the bank itself may be in the best position to evaluate their own risk.

¹⁶Note that in the case of a monoline insurer, we can think of bankruptcy as a ratings downgrade. The monoline business is based on having a better credit rating than the client in a process called *wrapping*. Without a good rating, it would not be profitable for firms to insure themselves with a monoline.

a payoff of zero if bankrupt.¹⁷ Therefore the expected payoff at time $t = 0$ for the IFI can be given by:

$$\Pi_{IFI}^{No\ Insurance} = \int_0^{\bar{R}_f} \theta dF(\theta). \quad (1)$$

Although the return is realized at $t = 2$, we assume that a perfect signal is sent about the return at $t = 1$. Therefore, at $t = 1$, the random variable θ represents the portfolio value if it could be costlessly liquidated at that time. However, the IFI's portfolio is assumed to be composed of both liquid and illiquid assets. In practice, we observe financial institutions holding both liquid (e.g., t-bills, money market deposits) and illiquid (e.g., loans, some exotic options, some newer structured finance products) investments on their books.¹⁸ Because of this, if the IFI wishes to liquidate some of its portfolio at $t = 1$, it will be subject to a liquidity cost which we discuss below in section II.C.

II.C. THE INSURANCE CONTRACT

We now introduce the means by which the bank is insured by the IFI. At $t = 0$, the bank requests an insurance contract in the amount of γ for one period of protection. Therefore, the insurance coverage is from $t = 0$ to $t = 1$. To begin, we assume that the bank contracts with one IFI who is in Bertrand competition. The IFI forms a belief b about the probability that the bank loan will default (so that a claim is made). In section III we will show how b is formed endogenously as an equilibrium condition of the model. In exchange for this protection, the IFI receives an insurance premium $P\gamma$, where P is the per unit price of coverage. The IFI chooses a proportion β of this premium to put in a liquid asset with a rate of return normalized to one in both $t = 1$ and $t = 2$. The liquid asset can be accessed in either time period. The remaining proportion $1 - \beta$ is put in an illiquid asset with an exogenously given rate of return of $R_I > 1$ which pays out at time $t = 2$.¹⁹ This asset can be thought of as a two period project that cannot be terminated early. It is this property that makes it illiquid. The key difference between these two assets is that the liquid asset is accessible at $t = 1$ when the underlying loan may default, whereas the illiquid asset is only available at $t = 2$.

For the remaining capital needed (net of the premium put in the liquid asset) if a claim is made, we assume that the IFI can liquidate its portfolio. Recall that the IFI's initial portfolio contains assets of possibly varying degrees of liquidity with return governed by F . To capture this, we assume that the IFI has a liquidation cost represented by the invertible function $C(\cdot)$ with

¹⁷All of the results of the paper still hold if the IFI receives a negative payoff in the event of failure, i.e., no limited liability. A supplement containing these results is available from the author upon request.

¹⁸If another bank acts as the IFI, it is obvious that many illiquid assets are on its books. However, this is also the very nature of many insurance companies and hedge funds businesses. In the case of insurance companies as the IFI, substantial portions of their portfolios may be in assets which cannot be liquidated easily (see Plantin and Rochet [2007]). In the case of hedge funds as the IFI, many of them specialize in trading in illiquid markets (see Brunnermeier and Pederson [2005] for example).

¹⁹Since R_I is fixed, we are assuming that the illiquid asset and the original portfolio are uncorrelated. Adding correlation would only complicate the analysis and would not change the qualitative results.

$C'(\cdot) > 0$, $C''(\cdot) \geq 0$, and $C(0) = 0$. The weak convexity of $C(\cdot)$ implies that the IFI will choose to liquidate the least costly assets first, but as more capital is required, it will be forced to liquidate illiquid assets at potentially fire sale prices.²⁰ $C(\cdot)$ takes as its argument the amount of capital needed from the portfolio, and returns a number that represents the actual amount that must be liquidated to achieve that amount of capital. This implies that $C(x) \geq x \forall x \geq 0$ so that $C'(x) \geq 1$. For example, if there is no cost of liquidation and if x_1 is required to be accessed from the portfolio, the IFI can liquidate x_1 to satisfy its capital needs. However, because liquidation may be costly in this model, the IFI must liquidate $C(x_1) = x_2 \geq x_1$ to obtain the needed x_1 .

At time $t = 1$, the IFI learns a valuation of its portfolio; however, the return is not realized until $t = 2$. This could be relaxed so that the IFI receives a fuzzy signal about the return; however, this would yield no further insight into the problem. Also at $t = 1$, a claim is made if the underlying borrower defaults. If a claim is made, the IFI can liquidate its portfolio to fulfil its obligation of γ . If the contract cannot be fulfilled, the IFI defaults. We assume for simplicity that if the IFI defaults, the bank receives nothing.²¹ At $t = 2$, the IFI and bank's return are realized. This setup implies that the uncertainty in the model is resolved at $t = 1$; however, a costly liquidation problem remains from $t = 1$ to $t = 2$. Figure II summarizes the timing of the model.

The expected payoff of the IFI can be written as follows.

$$\begin{aligned} \Pi_{IFI} = & (1 - b) \left[\int_{-P\gamma(\beta + (1 - \beta)R_I)}^{\bar{R}_f} (\theta + P\gamma(\beta + (1 - \beta)R_I)) dF(\theta) \right] \\ & + (b) \left[\int_{C(\gamma - \beta P\gamma)}^{\bar{R}_f} (\theta - C(\gamma - \beta P\gamma) + P\gamma(1 - \beta)R_I) dF(\theta) \right] \end{aligned} \quad (2)$$

The first term is the expected payoff when a claim is not made, which happens with probability $1 - b$ given the IFI's beliefs. The $-P\gamma(\beta + (1 - \beta)R_I)$ term in the integrand represents the benefit that engaging in these contracts can have: it reduces the probability of the IFI defaulting when a claim is not made. We assume that \underline{R}_f is sufficiently negative so that $P\gamma(\beta + (1 - \beta)R_I) < |\underline{R}_f|$. Since P and β are both bounded from above,²² it follows that this inequality is satisfied for a finite \underline{R}_f . This assumption ensures that the IFI cannot completely eliminate its probability of default in this state. Recall that before the IFI engaged in the insurance contract, it would be forced into insolvency when the portfolio draw was less than zero. However, if a claim is not made, it can receive a portfolio draw that is less than zero and still remain solvent (so long as the IFI's draw is greater than $-P\gamma(\beta + (1 - \beta)R_I)$). The payoff to the invested premium is also realized.

The second term is the expected payoff when a claim is made, which happens with probability b given by the IFI's beliefs. The term $C(\gamma - \beta P\gamma)$ represents the cost to the IFI of accessing the needed capital to pay a claim. Notice that premia placed in the illiquid asset are not available if a claim is made. Furthermore, the probability of default for the IFI increases in this case. To see this, notice

²⁰There is a growing literature on trading in illiquid markets and fire sales. See for example Subramanian and Jarrow (2001), and Brunnermeier and Pedersen (2005).

²¹This can be relaxed to allow partial recovery without changing the qualitative results.

²²This is true for β by construction and will be proven for P in Lemma 2.

that before engaging in the insurance contract, the IFI defaults if its portfolio draw is $\tilde{\theta} \in [\underline{R}_f, 0]$. After the insurance contract is sold, default occurs if the draw is $\tilde{\theta} \in [\underline{R}_f, C(\gamma - \beta P \gamma) > 0]$. The invested premium is realized; however, the liquid part is used to help pay off the claim. For the remainder of the paper, we will assume that (2) is globally concave in β .²³

As stated previously, counterparty risk is defined as the risk that the IFI defaults, conditional on a claim being made. Therefore, counterparty risk is represented in the model by $\int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} dF(\theta)$.

II.D. IFI BEHAVIOR

We now characterize the optimal investment choice of the IFI and the resulting market clearing price. The following lemma determines the optimal investment decision conditional on a belief (b) and a price (P). The IFI is shown to invest more in the liquid asset if it believes a claim is more likely. Let β_S^* (β_R^*) be the optimal choice of the IFI given it believes that the loan is safe (risky).

LEMMA 1 *The optimal investment in the liquid asset (β^*) is weakly increasing in the belief of the probability of a claim (b). Consequently, $\beta_R^* \geq \beta_S^*$.*

Proof. See Appendix A.

Note that the relationship in this proposition is strict when β^* attains an interior solution. The implicit expression for β^* can be found in the proof to this lemma as expression (19). The intuition behind this result is that when the IFI believes a claim is likely to be made, it minimizes costly liquidation by investing in the liquid asset. Alternatively, when the IFI believes a claim is unlikely to be made, it invests more in the illiquid asset which earns higher returns. It is easy to see that the optimal investment is conditional on a price P (where $P\gamma$ represents the insurance premium). We define P^* as the market clearing price. To characterize it, we use the assumption of Bertrand competition so that the IFI must earn zero profit from engaging in the insurance contract. The following lemma yields both existence and uniqueness of the market clearing price P^* .

LEMMA 2 *The market clearing price exists in the open set $(0, 1)$ and is unique.*

Proof. See Appendix A.

We now analyze the properties of the market clearing price P^* . The following lemma shows that as the IFI's belief about the probability of a claim increases, so too must the premium increase to compensate for the additional risk. Let P_S^* (P_R^*) be the market clearing price given the IFI believes that the loan is safe (risky).

LEMMA 3 *The market clearing price P^* is increasing in the belief of the probability of a claim (b). Consequently, $P_R^* > P_S^*$.*

²³The parameter space that supports this assumption is given in appendix A by equation (18).

Proof. See Appendix A.

This lemma yields the intuitive result that our pricing function $P(b)$ is increasing in b . We now begin a discussion on the equilibrium in the model.

III. EQUILIBRIUM BELIEFS

Akerlof (1970) showed how insurance contracts can be plagued by the ‘lemons’ problem. One underlying incentive in his model that generates this result is that the insured wishes only to minimize the premium paid. It is for this reason that high risk agents wish to conceal their type. Subsequent literature showed how the presence of a costly signaling device can allow a separating equilibrium to exist. What is new in our paper is that no such signaling device is needed to justify the existence of a separating equilibrium. We call the act of concealing one’s type for the benefit of a lower insurance premium the *premium effect*. In this section we show that this effect may be subdued in the presence of counterparty risk. This is done by demonstrating another effect that works against the premium effect that we call the *counterparty risk effect*. The intuition of this new effect is that if high risk (risky) agents attempt to be revealed as low risk (safe), a lower insurance premium may be obtained, but the following lemma shows that counterparty risk will increase.

LEMMA 4 *If b decreases, but the actual probability of a claim does not, counterparty risk rises whenever $\beta \in (0, 1]$.*

Proof. See Appendix A.

There are two factors that contribute to this result. First, Lemma 3 showed that as the perceived probability of a claim decreases, the premium also decreases and therefore leaves less capital available to be invested. Second, Lemma 1 showed that the IFI will put more in the illiquid asset as b decreases. Combining these two factors, the counterparty risk increases. The only case in which the counterparty risk will not rise is when the IFI is already investing everything in the illiquid asset, so that as b decreases, everything is still invested in the illiquid asset.

To analyze the resulting equilibria, we employ the concept of a Perfect Bayesian Nash Equilibrium (PBNE). At $t = 0$, after the bank is endowed with its project but before the price P^* is determined, a bank of type $i \in \{S, R\}$ (either risky or safe) sends a message $\mathcal{M} \in \{S, R\}$ (either risky or safe). Let the bank’s payoff be $\Pi(i, \mathcal{M})$ representing the profit that a type i bank receives from sending the message \mathcal{M} . Formally, an equilibrium in our model is defined as follows.

DEFINITION 1 *An equilibrium is defined as a portfolio choice β , a price P , and a belief b such that:*

1. b is consistent with Bayes’ rule where possible.
2. Choosing P , the IFI earns zero profit with β derived according to the IFI’s problem.

3. The bank chooses its message so as to maximize its expected profit.

To proceed we ask: is there a separating equilibrium in which both types are revealed truthfully? The answer without counterparty risk is *no*. The reason is that without counterparty risk, it is costless for the bank with a risky loan to imitate a bank with a safe loan. However, with counterparty risk, it is possible that both types credibly reveal themselves so that separation occurs. To begin, assume that the IFI's beliefs correspond to a separating equilibrium. Therefore, if $\mathcal{M} = S$ ($\mathcal{M} = R$) then $b = 1 - p_S$ ($b = 1 - p_R$). We now write the profit for a bank with a risky loan given a truthful report ($\mathcal{M} = R$).

$$\Pi(R, R) = p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{\bar{R}_f} dF(\theta) - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_R^* P_R^* \gamma)} dF(\theta) - \gamma P_R^* \quad (3)$$

The first term represents the expected payoff to the bank when a claim is not made. The second term represents the expected payoff on the insured portion of the loan when a claim is made and the IFI is able to fulfil it. Notice that the IFI's beliefs are such that the bank is risky. The third term represents the expected payoff when a claim is made and the IFI fails and so is unable to fulfil the contract. The final term is the insurance premium that the bank pays to the IFI. In a similar way, the profit of a risky bank who reports that it is safe ($\mathcal{M} = S$) is given as follows.

$$\Pi(R, S) = p_R R_B + \gamma(1 - p_R) \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{\bar{R}_f} dF(\theta) - \gamma(1 - p_R) Z \int_{\underline{R}_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) - \gamma P_S^* \quad (4)$$

We now find the condition under which a risky bank wishes to truthfully reveal its type.

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{\gamma(P_R^* - P_S^*)}_{\text{amount extra to be paid in insurance premia}} \quad (5)$$

From Lemmas 1 and 3 we know that $C(\gamma - \beta_R^* P_R^* \gamma) < C(\gamma - \beta_S^* P_S^* \gamma)$ and therefore the left hand side represents the counterparty risk that a risky bank saves by reporting truthfully. This is the *counterparty risk effect*. The right hand side represents the savings in insurance premia that the bank would receive by misrepresenting its type. This is the *premium effect*. The inequality (5) represents the key condition for the separating equilibrium to exist. Without counterparty risk, the left hand side must equal zero, and consequently, the risky agent will always misrepresent its type. We now turn to a bank with a safe loan and repeat the same exercise.

$$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^* - P_S^*}_{\text{amount to be saved in insurance premia}} \quad (6)$$

The left hand side represents the amount of counterparty risk that the bank will save if it conceals its type. The right hand side represents the amount of insurance premia that the bank will save if it reports truthfully. Therefore, when (5) and (6) hold simultaneously, this separating equilibrium exists. For an example of when these two conditions can hold, consider the case in which the safe loan is “very” safe. In particular, let $p_S \rightarrow 1$ which implies that $P_S^* \rightarrow 0$. Since $p_R < p_S$, P_R^* need not approach 0. We obtain the following expressions.

$$\underbrace{0}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_R^*}_{\text{amount to be saved in insurance premia}} \quad (7)$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_R^*}_{\text{amount extra to be paid in insurance premia}} \quad (8)$$

Inequality (7) is satisfied trivially when $p_R < 1$ since $P_R^* > 0$, while (8) is satisfied for Z sufficiently large. Recall that Z can be interpreted as the cost of counterparty failure when a claim is made. Therefore separation can be achieved when there is a high enough ‘penalty’ on the bank for taking on counterparty risk. The intuition is that a larger penalty forces the bank to internalize the counterparty risk more. As a result, more information is revealed in the market. This is a sense in which counterparty risk may be beneficial to the market, since it can help alleviate the possible adverse selection problem caused by asymmetric information.

Both costly signalling and cheap talk games (as in the case of this model) are notorious for having multiple equilibria (e.g., a babbling equilibrium always exists) so that the only way to narrow down the prediction is to use refinements. The majority of refinements developed for the PBNE, such as the intuitive criterion from Cho and Kreps (1987) have no bite in cheap talk games. To obtain uniqueness, we will employ the well known refinement criterion found in Farrell (1993); however, other criteria that work equally well are discussed in the proof to Proposition 1. Let $P_{1/2}^*$ be the market clearing price and $\beta_{1/2}^*$ be the optimal investment choice when the IFI does not update its beliefs ($b = \frac{1}{2}(2 - p_S - p_r)$). We now state the first major result of the paper.

PROPOSITION 1 *In the absence of counterparty risk, no separating equilibrium can exist. When there is counterparty risk, a unique equilibrium exists in which each bank type truthfully announces its loan risk, i.e., a separating equilibrium. Necessary and sufficient conditions for uniqueness are 1) the safe loan is relatively safe and 2) Z is large enough. Formally,*

$$\begin{aligned}
1. & \left(p_s : p_s \geq 1 - \frac{P_R^* - P_S^*}{(1+Z) \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{C(\gamma - \beta_R^* P_R^* \gamma)} dF(\theta)} \ \& \ p_s > 1 - \frac{P_{1/2}^* - P_S^*}{(1+Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \right) \\
2. & \left(Z : Z \geq \frac{P_R^* - P_S^*}{(1-p_R) \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{C(\gamma - \beta_R^* P_R^* \gamma)} dF(\theta)} - 1 \ \& \ Z > \frac{P_R^* - P_{1/2}^*}{(1-p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} - 1 \right).
\end{aligned}$$

Proof. See Appendix A.

This proposition shows that a moral hazard problem on the part of the insurer can alleviate a possible adverse selection problem on the part of the insured. The separating equilibrium corresponds to the case in which the *premium effect* dominates for the bank with a safe loan, while the *counterparty risk effect* dominates for the bank with a risky loan.

If either of the two conditions of Proposition 1 are not met, then one of two pooling equilibria may exist (we formalize them in the proof to Proposition 1). The first occurs when both the safe and risky bank report that they are safe. In this case, the *premium effect* dominates for both types so that the IFI does not update its prior beliefs. The second (and less intuitive) pooling equilibrium occurs when both the safe and risky bank report that they are risky. In this case, the *counterparty risk effect* dominates for both types. The parameter of interest in these two pooling equilibria is Z . For example, when Z is low, the bank is subjected to little penalty for the risk that the counterparty will default when a claim is made. This could be the case if a bank engages in credit risk transfer to satisfy regulatory requirements and is not concerned with the chance that it may not be protected.²⁴ Specifically, Z can be thought of as a proxy for the bank's aversion to counterparty risk. The less averse to that risk it is, the less it will be concerned with the welfare of its counterparty. If the following holds:

$$Z \leq \frac{P_R^* - P_{1/2}^*}{(1-p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} - 1, \tag{9}$$

then the second formal condition of Proposition 1 is violated and the first pooling equilibrium can exist.²⁵ In this case, both types prefer a lower insurance premium versus reduced counterparty risk. Alternatively, if the following holds:

$$Z \geq \frac{P_{1/2}^* - P_S^*}{(1-p_S) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} - 1, \tag{10}$$

then the first formal condition of Proposition 1 is violated and the second pooling equilibrium can exist. In this case, both types prefer less counterparty risk versus a lower premium.

²⁴For regulatory capital, Basel II requires a counterparty to have at least as good of a credit rating as the insured. However, recent financial events such as the collapse and bailout of AIG have taught us that even those financial firms that once had good credit ratings and who potentially act as insurers can fail.

²⁵Whether or not this pooling equilibrium exists depends on off the equilibrium path beliefs and is discussed in the proof to Proposition 1.

We now remove a key contracting imperfection to highlight the inefficiency in the IFI's investment choice and formally prove the existence of a moral hazard problem. Subsequently, we perform a welfare analysis linking the cost of moral hazard to the benefit of the separating equilibrium and to the cost of the traditional separation mechanism.

III.A. CONTRACT INEFFICIENCY

In this section, we imagine a planning problem wherein the planner can control the investment decision of the IFI. However, we maintain the IFI's beliefs and zero profit condition. We show that regardless of the beliefs of the IFI, the planner can always do better than is done in equilibrium by increasing the amount of capital put in the liquid asset. Therefore, this section will show that we can get closer to a first best allocation by removing this contracting imperfection, thereby highlighting the moral hazard problem. We denote the solution to the social planner's problem given any belief b as β_b^{sp} , with resulting price P_b^{sp} . We can now show that a moral hazard problem exists. The following proposition shows that the IFI chooses a β^* that is too small as compared to that of the planner's problem β^{sp} for any belief of the IFI. Consequently, the insurer moral hazard problem causes the level of counterparty risk in equilibrium to be too high (note that if the equilibrium outcome is characterized by full investment in the liquid asset $\beta^* = 1$, the social planner cannot choose $\beta^{sp} > 1$ so cannot improve the IFI's investment).

PROPOSITION 2 *Any competitive equilibrium characterized by an investment decision $\beta^* \in [0, 1)$ is inefficient.*

Proof. *See Appendix A.*

The intuition behind this result comes from two sources. First, since the social planning problem corresponds to maximizing the bank's payoff while keeping the IFI at zero profit, the bank strictly prefers to have the IFI invest more in the liquid asset. Second, the IFI must be compensated for this *individually* sub-optimal choice of β by an increase in the premium. Since P increases to compensate the IFI for the suboptimal investment (this is established in the proof to Proposition 2), counterparty risk falls (i.e., $\int_{R_f}^{C(\gamma - \beta P \gamma)} dF(\theta)$ decreases). In other words, the moral hazard problem on the part of the IFI is characterized by an inefficiency in the investment choice. The key restriction on the contracting space that yields this result is that the insurance premium is paid upfront and so the bank cannot condition its payment on an observed outcome. In the competitive equilibrium case, the bank knows that the IFI will invest too little into the liquid asset, and therefore lowers its payment accordingly. Although the limited liability, excess risk taking result of Jensen and Meckling (1976) is left unexplored, the inefficiency in our paper does not arise from this channel. Instead, the inefficiency arises because the IFI makes its decision based only on its own risk, and not the risk that it causes to the bank. The social planner forces the IFI to internalize the counterparty

risk it creates for the bank.²⁶

We now analyze the inefficiency caused by the moral hazard problem determined in Proposition 2 versus the welfare benefit of the separating equilibrium.

III.B. THE BENEFIT OF SEPARATION AND THE INEFFICIENCY FROM MORAL HAZARD

It is interesting to ask whether the separating equilibrium, by revealing more information to the market, can more than offsets the welfare loss in the market inefficiency due to non-contingent contracts. It is complicated and tedious to analyze general welfare results; however, it is possible to show that the result can go both ways. Through the use of a simple example, we demonstrate in this section that the benefit from separation can more than compensate for the inefficiency caused by moral hazard. Define total welfare as the sum of the expected profit of the IFI and the bank. We use the welfare in the pooling equilibrium as the benchmark case (W_{pool}). Let the welfare in the separating equilibrium be W_{sep} . The benefit of separation can then be found by taking $W_{sep} - W_{pool}$. Next, let the welfare in the social planners problem be W_{pool}^{sp} . Recall that the social planner eliminates the moral hazard by forcing the IFI to invest more in liquid assets. Let W_{pool}^{sp} represent the welfare when there is no moral hazard problem and the two bank types pool. The inefficiency from moral hazard is then given by $W_{pool}^{sp} - W_{pool}$. Therefore, the difference between the benefit from the separation of types and the inefficiency from moral hazard is given by \mathcal{W} as follows.

$$\mathcal{W} = W_{sep} - W_{pool}^{sp} \quad (11)$$

Therefore, if $\mathcal{W} > 0$, the benefit from separation more than offsets the inefficiency caused by the moral hazard. This inequality will hold in general when the following is satisfied (found by plugging expressions for W_{sep} and W_{pool}^{sp} into (11)).

$$\begin{aligned} & (1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^{sp} P_{1/2}^{sp} \gamma)} dF(\theta) - (P_R^* - P_{1/2}^{sp}) \\ & + (P_{1/2}^{sp} - P_S^*) - (1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{1/2}^{sp} P_{1/2}^{sp} \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) > 0. \end{aligned} \quad (12)$$

The first term represents the difference in counterparty risk for the risky type between the social planning pooling case and the separating equilibrium. The second term represents the difference in premia that the risky type must pay.²⁷ The third term represents the increased premium that the safe type pays in the social planning case as compared to the equilibrium case. Finally, the

²⁶It should be stressed that the separating equilibrium result can obtain in both the moral hazard and social planning cases. We focus on the more interesting case in which the moral hazard is present.

²⁷Note that in a typical problem without counterparty risk, transferable utility implies that prices should not affect the total utility (welfare). In this case however, the potential bankruptcy of the IFI implies a deadweight loss so that total welfare can be a function of prices.

fourth term represents the amount by which counterparty risk is decreased for the safe type in the social planning case as compared to the separating equilibrium. We can think of this expression as containing both bankruptcy and investment efficiency. The parameter Z represents the inefficiency caused by the bankruptcy of the IFI. For example, the first term in (12) represents the efficiency from the risky bank truthfully revealing itself. The second and third terms contain investment efficiency. To see this, note that the socially optimal investment level is inefficient from the point of view of the IFI. For example, we can write the third term as $(P_{1/2}^{sp} - P_{1/2}^*) - (P_S^* - P_{1/2}^*)$. The term $P_{1/2}^{sp} - P_{1/2}^*$ represents the additional amount that the bank must pay due to the individually sub-optimal choice that the social planner makes for the IFI, i.e., the investment inefficiency.

To see that (12) can hold, consider the following parametrization. Let $F(\theta)$ be the uniform distribution, $C(x) = x + \frac{1}{2}x^2$, $p_S = 0.1$, $p_{1/2} = 0.35$, $p_R = 0.6$, $R_I = 1.25$, $\bar{R}_f = 30$, $\underline{R}_f = -5$, $\gamma = 5$ and $Z = 1$. So that γ can exceed 1, we do not normalize the size of the bank's loan to 1. We computationally implement the model and obtain the following results: $\beta_R^* = 1$, $P_R^* = 0.703$, $\beta_{1/2}^* = 1$, $P_{1/2}^* = 0.511$, $\beta_S^* = 0$ and $P_S^* = 0.206$. Since $\beta_{1/2}^* = 1$, it follows that $\beta_{1/2}^{sp} = 1$ and $P_{1/2}^{sp} = P_{1/2}^* = 0.511$. Intuitively, since the IFI already invests everything in the liquid asset without intervention from the social planner, the planner cannot do any better and thus makes the same investment choice.

We first verify that the separating equilibrium exists in this example. Recall that condition (31) given in Appendix A is required to hold for separation to take place. We reproduce this condition here for ease of reference.

$$\frac{P_R^* - P_S^*}{(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \leq 1 + Z \leq \frac{P_R^* - P_S^*}{(1 - p_S) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \quad (13)$$

Using our example we find that the left hand side equals 1.94 while the far right hand side equals 11.65. Since $1 + Z = 2$, it follows that (13) is satisfied so that separation exists. We are left to verify that the welfare effect of separation is positive. We obtain the following results.

$$(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_{1/2}^{sp} P_{1/2}^{sp} \gamma)} dF(\theta) - (P_R^* - P_{1/2}^{sp}) = -0.0943 \quad (14)$$

$$(P_{1/2}^{sp} - P_S^*) - (1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{1/2}^{sp} P_{1/2}^{sp} \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) = 0.2355 \quad (15)$$

It follows that (12) holds by substituting in the values obtained from (14) and (15). Therefore, this example shows a case in which the ability to separate more than compensates for the loss due to moral hazard. Note that the sign of (14) means that the risky bank does better in the social planning case. This occurs because the investment decision of the IFI in this example is the same for the risky type in both the social planning and separating equilibrium cases (i.e., $\beta_{1/2}^{sp} = \beta_R^* = 1$). The only difference between the two cases is the price (premium); the risky bank must pay more in the separating equilibrium. This increases the total amount invested in the liquid asset by the IFI. This leads to a decrease in counterparty risk; however, this benefit is overshadowed by the actual

increase in price (premium).

As one would expect, this example shows that the moral hazard problem has a cost; however, it also shows that it has a benefit as one may not have expected. Policy makers need to consider the possibility that controlling the investments of a financial institution can affect the incentives of those who contract with it. One way that a policy maker can avoid eliminating the incentives for truthful revelation is to allow the IFI to have some control over its investment decision. For example, the IFI could determine the level of credit risk, while the government/regulator could determine the investment conditional on a level of credit risk. This resembles one of the key features of the Basel II banking regulatory framework: to give banks more control over determining their own credit risk. This provision could be welfare improving, provided that the bank (acting as an IFI) has sufficient control over its investment decision so that the separating equilibrium can still exist.

We now compare the cost of the traditional separation mechanism to the cost that moral hazard imposes on the counterparty risk mechanism of separation.

III.C. THE COST OF MORAL HAZARD IN SEPARATION VERSUS THE COST OF THE TRADITIONAL SEPARATION MECHANISM

In what follows, we describe the basic features of the welfare comparison and intuitively discuss the results. A detailed analysis can be found in Appendix B. To compare the relative costs, we first need to discuss the traditional separation mechanism. In general, the traditional method of separation involves the safe bank reducing its coverage. In equilibrium, it must reduce its coverage enough so that the risky bank does not imitate. Let $\phi\gamma$ amount of coverage, where $\phi \in [0, 1]$. One can think of this as reduction in coverage with certainty. On the other hand, when we consider the counterparty risk mechanism of separation in this paper, this can be thought of as an expected reduction in coverage. The reason for this is that the bank does not reduce the contract size as in the traditional separation mechanism, but rather the expected coverage is reduced because of the risk that a claim may not be fulfilled. We refer to the counterparty risk case as the situation in which separation is achieved through the existence of the moral hazard problem (the focus of this paper). Let the traditional separating case be the situation in which there is no moral hazard and separation is achieved by the bank reducing its coverage. To maintain consistency in the two separation cases, we assume that a unit of increased counterparty risk is equivalent in terms of cost to the bank to a unit of decreased coverage in the traditional separating mechanism. To understand this assumption we use the following example. Let the probability that the IFI is insolvent when a claim is made be $1 - \wp$. The expected coverage is then given by $\wp\gamma$ and the expected cost incurred by the bank is $(1 - \wp)\gamma Z$. If there is no counterparty risk, and the bank chooses the same coverage, $\phi = \wp$, then the cost to the bank is the same, $(1 - \phi)\gamma Z = (1 - \wp)\gamma Z$. Next, consider the premium paid in both separating cases. For consistency, we assume that if the amount of protection is equal in the two cases (e.g., $\wp = \phi$), then the premium must also be the same. This condition is stated mathematically in Appendix B, Assumption 2. To make welfare comparisons, let the benchmark case be one in which the social planner can observe the bank type and control the IFI's

investment decision. Therefore, there is no moral hazard in this case. To make the two separating cases comparable, we assume that asymmetric information is present in the traditional separating mechanism. Furthermore, we assume that counterparty risk remains unchanged in the traditional separating case for each bank type from the benchmark case. In other words, the bank does not suffer any additional counterparty risk beyond that of the base case. This assumption is required because there is counterparty risk even in the absence of moral hazard in the counterparty risk case. To ensure that the bank has a meaningful choice in the traditional separation mechanism, assume that the benchmark level of counterparty risk is not sufficient to achieve separation (the condition that supports this is formalized in Appendix B, Assumption 1). In other words, the safe bank must reduce its coverage to separate.

As in the previous section, we can define total welfare as the sum of the expected payoff of the IFI and bank. Since the IFI will always earn zero profit in equilibrium, we can focus on the bank. It turns out that the counterparty risk mechanism of separation is at least as costly as the traditional mechanism (Proposition 4, Appendix B). The intuition behind this result is as follows. The risky bank always receives at least as much coverage in the traditional setup than in the counterparty risk case. The reason for this is that the risky bank is not subject to moral hazard in the traditional setup while it is in the counterparty risk case. The safe bank benefits from an increased flexibility of coverage. In both separating mechanisms, the safe type must bear more risk than the risky type to separate. In the counterparty risk case, the difference in coverage (counterparty risk) between the safe and risky type is given by $\int_{C(\gamma-\beta_R^*P_R^*\gamma)}^{C(\gamma-\beta_S^*P_S^*\gamma)} dF(\theta)$. Since this expression is determined by the IFI's problem, the safe bank has no control over how much it reduces its coverage to achieve separation. In the traditional separation mechanism however, the amount by which the safe bank reduces its coverage is controlled by the safe type directly. With this control, the safe bank reduces its coverage by just enough so that the risky bank truthfully reveals its type. Therefore, the safe bank may receive more protection while still achieving separation in the traditional separating case.

It is important to note that if the moral hazard problem of the IFI exists, then the bank has no choice but to use the counterparty risk mechanism. In other words, it would not be worthwhile for the safe bank to reduce its coverage further since the IFI will already know its type due to the counterparty risk mechanism. Therefore, the counterparty risk mechanism can prevail regardless of whether it would be welfare improving to use the traditional separation mechanism. There is an upside however, as the two separation mechanisms can act as complements to each other. For example, consider the case in which Z is not sufficiently large to yield separation (so that (5) is violated). In this case, both bank types report that they are safe. With both separation mechanisms available, the safe bank could reduce its coverage so that this reduction, plus the increased counterparty risk (of being revealed as safe), is sufficiently unattractive to the risky bank that it truthfully reveals its type.

With the welfare analysis complete, we now generalize the base model to the case in which there are multiple insured parties.

IV. MULTIPLE BANKS

In this section, we analyze the case of multiple banks and one insurer. We assume there are a measure $M < 1$ of banks. This assumption is meant to approximate the case in which there are many banks, and the size of each individual bank's insurance contract is insignificant for the IFI's investment decision. Each bank requests an insurance contract of size γ . At time $t = 0$, each bank receives both an aggregate and idiosyncratic shock (both observed privately by the banks) which assigns them a probability of default. As in the case when there was only one bank, the return on the loan is assumed to be R_B if it succeeds and 0 if it does not. We define the idiosyncratic shock by the random variable X and let it be uniformly distributed over $[0, M]$. The CDF can then be written as follows (where x is a real number).

$$\Psi(x) = \begin{cases} 0 & \text{if } x \leq 0 \\ \frac{x}{M} & \text{if } x \in (0, M) \\ 1 & \text{if } x \geq M \end{cases}$$

Next, denote the aggregate shock as q_A and let it take the following form:

$$q_A = \begin{cases} s & \text{with probability } \frac{1}{2} \\ r & \text{with probability } \frac{1}{2}, \end{cases}$$

where $0 < s < r < 1 - M$. We assume that the probability of default of bank i is $q_i = q_A + X_i$.²⁸ We will refer to the aggregate shock as either (s)afe or (r)isky. We can think of this procedure as putting the banks in one of two intervals, either $[s, s + M]$ or $[r, r + M]$. To obtain existence of a separating equilibrium, no restrictions are required relating $s + M$ to r other than what is implied by the assumption that $0 < s < r < 1 - M$. In Proposition 3, it will be shown that uniqueness for a subset of the parameter space can be established if $r > s + M$. We write the conditional distributions of bank types as $\mu(q_i : q_i \leq x | q_A = s) = \Psi(x - s)$ and $\mu(q_i : q_i \leq x | q_A = r) = \Psi(x - r)$. It follows that $\Psi(x - s)$ first order stochastically dominates $\Psi(x - r)$ since $\Psi(x - s) \geq \Psi(x - r) \forall x$. Note that lower draws refer to a lower probability of default; a 'better' outcome.

IV.A. THE IFI'S PROBLEM

Because of the asymmetric information problem, the IFI does not know ex-ante whether the aggregate shock was $q_A = s$ or $q_A = r$. However, the IFI does know that the aggregate shock hits all the banks in the same way.²⁹ Therefore, if a subset of the banks can successfully reveal their aggregate type, this reveals the aggregate shock for the rest of them.

If solvent, the IFI must pay γ to each bank whose loan defaults. In Lemma 6 we will show

²⁸Note that in the base model we referred to p as a probability of success, whereas here we refer to q as a probability of failure. We make this notational change because it is more intuitive in this section to have probabilities of failure when we introduce the IFI's beliefs over the measure of defaults. Of course, the simple relationship $p = 1 - q$ holds.

²⁹This assumption can be relaxed to allow the banks to receive correlated draws from a distribution.

that there can be no separation of types within the idiosyncratic shock. Therefore, given a fixed realization of the aggregate shock, each bank pays the same premium P .³⁰ We assume that the IFI has the same choice as in section II.C, so that it invests β in the liquid storage asset and $(1 - \beta)$ in the illiquid asset with return R_I . To represent the IFI's beliefs, let Y denote the measure of defaults. Furthermore, let $b(y) = \text{prob}(Y \leq y)$ be the beliefs over the measure of defaults defined over $[0, M]$. It follows that $\Psi(x - s) \geq \Psi(x - r) \forall x$ implies $b(y|q_A = s) \geq b(y|q_A = r) \forall y$. In other words, first order stochastic dominance is preserved. Since each bank insures γ , the total size of contracts insured by the IFI is: $\int_0^M \gamma d\Psi(x) = M\gamma$. Let 'MB' denote 'Multiple Banks' so that the IFI's payoff can now be written as follows.

$$\begin{aligned} \Pi_{IFI}^{MB} = & \underbrace{\int_0^{\beta PM} \left[\int_{-PM\gamma(\beta+(1-\beta)R_I)+y\gamma}^{\bar{R}_f} (\theta + (\beta + (1 - \beta) R_I) PM\gamma - y\gamma) dF(\theta) \right] db(y)}_{\text{Term 1}} \\ & + \underbrace{\int_{\beta PM}^M \left[\int_{C(y\gamma-\beta PM\gamma)}^{\bar{R}_f} (\theta - C(y\gamma - \beta PM\gamma) + (1 - \beta)R_I PM\gamma) dF(\theta) \right] db(y)}_{\text{Term 2}} \quad (16) \end{aligned}$$

The first term represents the case in which the IFI puts sufficient capital in the liquid asset so that there is no need to liquidate its portfolio to pay claims. This happens if a sufficiently small measure of banks make claims. Since the IFI receives $PM\gamma$ in insurance premia, it puts $\beta PM\gamma$ into the liquid asset. It follows that if less than $\beta PM\gamma$ is needed to pay claims (i.e., less than βPM banks make a claim), portfolio liquidation is not necessary. The second term represents the case in which the IFI must liquidate its portfolio if a claim is made. This happens if the amount it needs to pay in claims is greater than $\beta PM\gamma$. $C(y\gamma - \beta PM\gamma) + \beta PM\gamma$ represents the total cost of claims, where $y\gamma - \beta PM\gamma$ is the total amount of capital the IFI needs to liquidate from its portfolio. As in the base model, we assume that if a claim is made and the IFI defaults, the banks do not receive any payment from the IFI. Also as in the base model, we restrict our attention to the case in which Π_{IFI}^{MB} is globally concave in β .

The following lemma derives the optimal β^* and proves that counterparty risk is less severe when the IFI believes that the loans are more risky.

LEMMA 5 *For a given aggregate shock, there is less counterparty risk when the IFI's beliefs put more weight on the aggregate shock being risky ($q_A = r$) as opposed to it being safe ($q_A = s$).*

Proof. See Appendix A.

The intuition for this result is similar to that of Lemma 4. If the IFI believes that the pool of loans is risky, it is optimal to invest more in the liquid asset. This happens because the expected

³⁰We assume the IFI must earn zero profit in equilibrium so that a market clearing price can be determined; however, we are not concerned with pinning down this price in this section.

number of claims is higher in the risky case. As such, the IFI wishes to prevent costly liquidation by investing more in assets that will be readily available when claims are made.

We now give the conditions under which the IFI's beliefs ($b(y)$) are formed.

IV.B. EQUILIBRIUM BELIEFS

IV.B.1. NO AGGREGATE SHOCK

To analyze how the beliefs of the IFI are formed, consider the case where there is no aggregate shock. Since there is no aggregate uncertainty, the IFI's optimal investment choice remains the same regardless of whether it offers a pooling price or individual separating prices.³¹ It follows that since an individual bank's choice will have no effect on counterparty risk, only the *premium effect* is active. It is for this reason that a separating equilibrium in the idiosyncratic shock cannot exist. To see this, assume that each bank reveals its type truthfully. Now consider the bank with the highest probability of default, call it bank M . Since by revealing truthfully it pays the highest insurance premium, it can lie about its type without any effect on counterparty risk, and obtain a better premium (and consequently a higher payoff). The following lemma formalizes.

LEMMA 6 *There can be no separating equilibrium in which the idiosyncratic shock is revealed.*

We now introduce the aggregate shock and show that separation of aggregate types can occur.

IV.B.2. AGGREGATE AND IDIOSYNCRATIC SHOCKS

Each individual bank now receives both an aggregate and an idiosyncratic shock. We know that if one bank is able to successfully reveal its aggregate shock, then the aggregate shock is revealed for all banks.³² The following proposition shows that there exists a parameter range in which a unique equilibrium characterized by separation of aggregate types can exist. As in the case of the base model with only one bank, cheap talk games are notorious for multiple equilibria (e.g., a babbling equilibrium always exists). To refine our prediction and obtain uniqueness, we employ the same refinement criterion of Farrell (1993) and discuss other criteria that will work equally well in the proof.

PROPOSITION 3 *There exists a separating equilibrium in which the aggregate shock is revealed. If $r > s + M$, then there exists a parameter range in which a unique equilibrium can be supported in which the aggregate shock is revealed, i.e., a separating equilibrium.*

Proof. *See Appendix A.*

³¹To see this, note that with no aggregate risk, the IFI knows the average quality of banks and will use that to make its investment decision. Any bank claiming that it received the lowest idiosyncratic shock will not change the IFI's beliefs about the average quality.

³²Note that in the proof to Proposition 3 (which is stated at the end of this section), we detail the issue of a measure zero bank affecting the IFI's beliefs and discuss how to handle it.

The insight from the above proposition follows from an individual bank's ability to affect the IFI's investment choice (through the IFI's beliefs). If a bank could only reveal its own shock, its premium would be insignificant to the IFI's investment decision. However, since by successfully revealing itself, a bank also reveals for the other banks, an individual's problem can have a significant effect on the IFI's investment choice. Note that the parameter range that supports a unique separating equilibrium is given by (45) and (48). These conditions are similar to the one bank case: Z sufficiently high, and the safe aggregate shock sufficiently low. If either condition (43) or (44) is met, then a pooling equilibrium exists in which no information about the aggregate shock is revealed.

We now turn to a brief discussion on the robustness of the results.

V. ROBUSTNESS

In this section we address five assumptions of the model. First, we consider the assumption that the bank can costlessly monitor. In particular, we discuss how the traditional moral hazard problem would manifest itself in our model. Second, we consider an enlarged type space and discuss why, in addition to the separating and pooling equilibria, there can be partial information revelation. Third, we discuss the implications of the bank being able to contract with multiple IFI's. Fourth, we detail how to relax the assumption that the IFI's portfolio is uniformly distributed. Finally, we address the assumption that there is only a liquid and an illiquid investment choice as well as the assumption that the IFI's initial portfolio is fixed.

The Traditional Moral Hazard Problem

The model can be extended to include a moral hazard problem on the part of the insured. This moral hazard arises by assuming that the insured can affect the probability that a claim is made. If we use the example of a bank insuring itself on one of its loans, the literature typically assumes that a bank possesses a proprietary monitoring technology (due to a relationship with the borrower). It is straightforward to see that if the bank is fully insured, it may not have the incentive to monitor the loan and, consequently, the probability of default of that loan could rise. The new moral hazard introduced in this paper may increase the desire of the insured to monitor. This happens because counterparty risk forces the bank to internalize some of the loan default risk which it otherwise would not. More importantly, it can be shown that the addition of this insured moral hazard problem does not affect the results of the paper.

Separating Equilibrium Result

The separating equilibrium result is no doubt stark. If we allow a bank to reveal a portion of its risk (in the case of multiple banks, a portion of their aggregate risk), we can create a setting in which partial information revelation is achieved. Consider three loan (or bank) types: one very risky, one moderately risky, and one safe. In this setup, a parametrization will exist in which all

types report that they are safe, another in which all types report that they are very risky and another in which all types report truthfully. The first two represent complete pooling equilibria while the third represents full separation. There are other equilibria in which, for example, the very risky type reports that it is moderately risky, while the other two types report truthfully. Here the very risky type wishes to reveal that it is not safe; however, it does not wish to disclose the true extent of its risk. In this way, it is possible that a type wishes to disclose some but not all of its risk. This would allow a range of equilibria to exist that fall between full separation and complete pooling in terms of information revelation. As in the model in the current paper, what would determine how much information revelation occurs is the banks aversion to counterparty risk (Z).

Multiple IFI's

We could allow the bank to insure with multiple IFI's. In this case, each IFI would solve its problem with a reduced insurance liability (γ). It can be shown that a decreased liability will cause each IFI to act riskier than it would if there was only one of them. Consequently, for a given equilibrium outcome, if we compare the case with one IFI to the multiple IFI's case, the total counterparty risk that the bank must suffer may not decrease as much as one might expect. In extreme cases, it may not decrease at all. This result is dependant on the degree of correlation of the IFI's portfolios.

Uniform Distribution Assumption

The uniform assumption can be relaxed to a general distribution, provided that it satisfies some conditions. For the moral hazard to obtain, the FOC and SOC conditions must be satisfied with the new distribution (see the proof to Lemma 1). For the separating equilibrium result to be possible, the region $[0, C(\gamma - \beta P\gamma)]$ must have positive mass. The reason is that if there were no mass in this region, the IFI's decision would have no effect on counterparty risk. To see this notice that (5) and (6) cannot be simultaneously satisfied if $\int_0^{C(\gamma - \beta P\gamma)} dF(\theta) = 0$.

Investment Choice and Fixed Initial Portfolio Assumption

It is not crucial that the IFI's investment choice is between only a liquid and an illiquid asset. We could alter the environment and have the choice be between a risky and riskless asset and maintain the qualitative results of the paper. In the current model, the IFI invests more in the illiquid asset when it believes it is less likely that a claim will be made. The reason for this is that conditional on a claim being made, the liquid asset is the most beneficial. The same intuition can hold when the choice is between a risky and riskless asset. The risky asset may provide the IFI with a higher expected return; however, conditional on a claim being made, the riskless asset decreases the IFI's probability of bankruptcy more than the risky asset. The IFI then invests more in the riskless asset if it believes it is likely a claim will be made, and the risky asset otherwise.

In addition to the premium investment choice, the model assumes for simplicity that the initial portfolio is fixed. We can imagine a richer model wherein the IFI could also change its initial portfolio. The amount that the IFI would change the portfolio would depend on the portfolios size

relative to the potential losses from claims. In this richer environment, as discussed above, we could include assets of varying degrees of liquidity and/or risk (as were assumed to be present in the initial portfolio). As in the simpler setup we use in our model, the decision of the IFI as to how risky or illiquid it would invest would depend on its expected loss due to claims. The results would then be qualitatively similar to the current setup.

Note that the insurer's investment problem deals with the asset side of the IFI's balance sheet. We could have modelled this problem from the liability side. For example, when AIG required government assistance, it was revealed that it had issued CDS contracts worth about 44% of its assets.³³ This would require a richer model; however, we can imagine the same basic risk tradeoff: the less risky the IFI expects contracts to be, the more risk it would like to take on.

VI. CONCLUSION

In a setting in which insurers can fail, we posit a new moral hazard problem that can arise in insurance contracts. If the insurer suspects that the contract is safe, it puts capital into less liquid assets which earn higher returns. The downside of this is that when a claim is made, the insurer is less likely to be able to fulfil the contract. We demonstrate that the insurer's investment choice is inefficiently illiquid. The presence of this moral hazard is shown to allow a unique separating equilibrium to exist wherein the insured freely and credibly relays its superior information. In other words, the new moral hazard problem can alleviate the possible adverse selection problem.

The results of the base model require the contract to be large enough to affect the insurer's investment decision. We relax this assumption and allow there to be a collection of insured parties, each with a contract size that is insignificant to the insurer's investment decision. We show that our moral hazard problem still exists, and can obtain the separating equilibrium result when there is private aggregate risk.

SCHOOL OF ACCOUNTING AND FINANCE, UNIVERSITY OF WATERLOO

VII. APPENDIX

VII.A. APPENDIX A

Proof Lemma 1

We maximize the IFI's additional profit from the insurance contract. The objective function can

³³Forbes (www.forbes.com). March 03, 2009. "Who's Afraid Of Credit Default Swaps?". Thank you to a referee for pointing this out.

be obtained by subtracting (1) from (2). With a slight abuse of notation, we denote this by Π_{IFI} .

$$\begin{aligned} \Pi_{IFI} = & (1-b) \left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^0 \theta dF(\theta) + \int_{-P\gamma(\beta+(1-\beta)R_I)}^{\bar{R}_f} P\gamma(\beta+(1-\beta)R_I) dF(\theta) \right] \\ & + b \left[- \int_0^{C(\gamma-\beta P\gamma)} \theta dF(\theta) + \int_{C(\gamma-\beta P\gamma)}^{\bar{R}_f} (-C(\gamma-\beta P\gamma) + P\gamma(1-\beta)R_I) dF(\theta) \right] \end{aligned}$$

Using the assumption that $F(\theta)$ is distributed uniform over the interval $[\underline{R}_f, \bar{R}_f]$, we solve for the optimal choice of β for the IFI, given b and P .

$$\begin{aligned} \max_{\beta \in [0,1]} & \frac{(1-b)}{\bar{R}_f - \underline{R}_f} \left[-\frac{(P\gamma(\beta+(1-\beta)R_I))^2}{2} + (\bar{R}_f + P\gamma(\beta+(1-\beta)R_I)) P\gamma(\beta+(1-\beta)R_I) \right] \\ & + \frac{b}{\bar{R}_f - \underline{R}_f} \left[-\frac{(C(\gamma-\beta P\gamma))^2}{2} + (\bar{R}_f - C(\gamma-\beta P\gamma)) (-C(\gamma-\beta P\gamma) + (1-\beta)R_I P\gamma) \right] \end{aligned}$$

We obtain the following first order equation:

$$\begin{aligned} 0 = & \frac{bP\gamma}{\bar{R}_f - \underline{R}_f} [\bar{R}_f C'(\gamma - \beta P\gamma) - \bar{R}_f R_I - C'(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma)] \\ & + \frac{bP\gamma}{\bar{R}_f - \underline{R}_f} [C'(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I + C(\gamma - \beta P\gamma)R_I] \\ & - \frac{(1-b)P\gamma}{\bar{R}_f - \underline{R}_f} (R_I - 1) [\bar{R}_f + (\beta + (1-\beta)R_I)]. \end{aligned} \quad (17)$$

To ensure a maximum, we take the second order condition and show the inequality that must hold.

$$\begin{aligned} & (1-b)(R_I - 1)^2 + bP\gamma [-\bar{R}_f C''(\gamma - \beta P\gamma) + C''(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma)] \\ & + bP\gamma [C'(\gamma - \beta P\gamma)C'(\gamma - \beta P\gamma) - C''(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I - 2C'(\gamma - \beta P\gamma)R_I] < 0 \end{aligned} \quad (18)$$

Simplifying and plugging the boundary conditions for β into the FOC (17), we now derive the optimal proportion of capital invested in the liquid asset as an implicit function.

$$\left\{ \begin{array}{ll} \beta^* = 0 & \text{if } b \leq b^* \\ 0 = b [C'(\gamma - \beta P\gamma) (\bar{R}_f + P\gamma(1-\beta)R_I - C(\gamma - \beta P\gamma)) - R_I (\bar{R}_f - C(\gamma - \beta P\gamma))] \\ \quad - (1-b)(R_I - 1) [\bar{R}_f + (\beta + (1-\beta)R_I)] & \text{if } b \in (b^*, b^{**}) \\ \beta^* = 1 & \text{if } b \geq b^{**} \end{array} \right. \quad (19)$$

$$\text{where } b^* = \frac{(R_I - 1)(\bar{R}_f + R_I)}{(R_I - 1)(\bar{R}_f + R_I) + \bar{R}_f C'(\gamma) - \bar{R}_f R_I - C'(\gamma)C(\gamma) + C'(\gamma)P\gamma R_I + C(\gamma)R_I},$$

$$\text{and } b^{**} = \frac{(R_I - 1)(\bar{R}_f + 1)}{(R_I - 1)(\bar{R}_f + 1) + \bar{R}_f C'(\gamma - P\gamma) - \bar{R}_f R_I - C'(\gamma - P\gamma)C(\gamma - P\gamma) + C(\gamma - P\gamma)R_I}.$$

We now show that the optimal proportion of capital invested in the liquid asset is increasing in b

by finding $\frac{\partial \beta}{\partial b}$ from the FOC. First, define:

$$\begin{aligned} A &= (1-b)(R_I - 1)^2 + bP\gamma [-\bar{R}_f C''(\gamma - \beta P\gamma) + C''(\gamma - \beta P\gamma)C(\gamma - \beta P\gamma)] \\ &\quad + bP\gamma [C'(\gamma - \beta P\gamma)C'(\gamma - \beta P\gamma) - C''(\gamma - \beta P\gamma)P\gamma(1-\beta)R_I - 2C'(\gamma - \beta P\gamma)R_I] \\ &< 0 \end{aligned} \tag{20}$$

where the inequality (20) follows from the SOC. Assuming an interior solution and rearranging for $\frac{\partial \beta}{\partial b}$ yields the following.

$$\begin{aligned} \frac{\partial \beta}{\partial b} &= \frac{-(R_I - 1)(\bar{R}_f + (\beta + (1-\beta)R_I)) - A_1}{A} \\ &> 0 \end{aligned} \tag{21}$$

Where $A_1 = C'(\gamma - \beta P\gamma)(\bar{R}_f + P\gamma(1-\beta)R_I - C(\gamma - \beta P\gamma)) - R_I(\bar{R}_f - C(\gamma - \beta P\gamma))$. For the FOC to hold, it must be the case that $A_1 > 0$. The inequality (21) then follows. ■

Proof of Lemma 2

Step 1: Existence

As was done in the proof to Lemma 1, we remove the payoff the IFI receives before issuing the contract (as the zero profit condition must be implemented on only the additional payoff from the insurance contract). By rearranging (2) and subtracting $\int_0^{\bar{R}_f} \theta dF(\theta)$, we find a P^* that satisfies the following:

$$\begin{aligned} 0 &= (1-b) \left[\int_{-P\gamma(\beta+(1-\beta)R_I)}^0 \theta dF(\theta) + \int_{-P\gamma(\beta+(1-\beta)R_I)}^{\bar{R}_f} P\gamma(\beta + (1-\beta)R_I) dF(\theta) \right] \\ &\quad + (b) \left[- \int_0^{C(\gamma-\beta P\gamma)} \theta dF(\theta) + \int_{C(\gamma-\beta P\gamma)}^{\bar{R}_f} (-C(\gamma - \beta P\gamma) + P\gamma(1-\beta)R_I) dF(\theta) \right] \\ &\quad + \int_0^{\bar{R}_f} \theta dF(\theta) - \int_0^{\bar{R}_f} \theta dF(\theta) \end{aligned} \tag{22}$$

Consider $P^* \leq 0$. In this case, the IFI earns negative profits. To see this, notice that the first term is negative (the first part is zero, while the second is negative) while the second term is also negative (both parts are negative). Therefore, it must be that $\Pi_{IFI}(\beta^*, P^* \leq 0) < 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium.

Next, consider $P^* \geq 1$, and $\beta = 1$ (not necessarily the optimal value). In this case, the first term on the right hand side of (22) is strictly positive and the second term is zero since $(C(0) = 0)$. Since β^* is optimal, it can yield no less profit than $\beta = 1$ and therefore, $\Pi_{IFI}(\beta^*, P^* \geq 1) > 0$. This contradicts the fact that $\Pi_{IFI}(\beta^*, P^*) = 0$ in equilibrium. Therefore, if it exists, $P^* \in (0, 1)$.

To show that P^* exists in the interval $(0, 1)$, we differentiate the right hand side of (22) (using the assumption that $f(\cdot)$ is uniformly distributed) to show that profit is strictly increasing in P .

$$\begin{aligned}
\frac{\partial \Pi_{IFI}}{\partial P} &= \frac{b}{\bar{R}_f - \underline{R}_f} [(\bar{R}_f - C(\gamma - \beta P \gamma)) (C'(\gamma - \beta P \gamma) \beta \gamma + (1 - \beta) R_I \gamma)] \\
&\quad + \frac{b}{\bar{R}_f - \underline{R}_f} [C'(\gamma - \beta P \gamma) \beta \gamma (1 - \beta) R_I P \gamma] \\
&\quad + \frac{(1 - b)}{\bar{R}_f - \underline{R}_f} (\bar{R}_f + P \gamma (\beta + (1 - \beta) R_I)) \gamma (\beta + (1 - \beta) R_I) \\
&> 0
\end{aligned} \tag{23}$$

Where the inequality (23) follows because $\bar{R}_f \geq C(\gamma - \beta P \gamma)$. Therefore, since profit is negative when $P^* \leq 0$ and positive when $P^* \geq 1$, and since profit is a (monotonically) increasing function of P^* , profit must equate to zero within $P^* \in (0, 1)$.

Step 2: Uniqueness

Assume the following holds: $\Pi_{IFI}(\beta^*, P_1^*) = 0$. Since we have already shown that profit is a strictly increasing function of P^* , then $P_2^* > P_1^*$ ($P_2^* < P_1^*$) implies $\Pi_{IFI}(\beta^*, P_2^*) > 0$ ($\Pi_{IFI}(\beta^*, P_2^*) < 0$). Therefore, $\Pi_{IFI}(\beta^*, P_2^*) = 0$ implies $P_1^* = P_2^*$ must hold, so our price is unique. ■

Proof of Lemma 3

From the envelop theorem, we can ignore the effect that changes in b have on β when we evaluate the payoff at β^* . Plugging $\beta = \beta^*$ into (2) and taking the partial derivative with respect to b yields:

$$\begin{aligned}
\frac{\partial \Pi_{IFI}}{\partial b} \Big|_{\beta=\beta^*} &= - \left[\int_{-P\gamma(\beta^*+(1-\beta^*)R_I)}^{\bar{R}_f} (\theta + P\gamma(\beta^* + (1 - \beta^*)R_I)) dF(\theta) \right] \\
&\quad + \left[\int_{C(\gamma-\beta^*P\gamma)}^{\bar{R}_f} (\theta - C(\gamma - \beta^* P \gamma) + P\gamma(1 - \beta^*)R_I) dF(\theta) \right] \\
&< 0
\end{aligned} \tag{24}$$

The sign of $\frac{\partial \Pi_{IFI}}{\partial b} \Big|_{\beta=\beta^*}$ is negative because the absolute value of the first term is greater than the second. To see this, notice that the limits of integration of the first term are wider as well as the integrand is greater than that of the second term. Intuitively, as b increases it is more likely that the IFI will have to pay a claim, and more likely that it will have to liquidate its portfolio. Both of these activities are costly. Since the envelop theorem is a local condition and does not hold for large changes in b , it serves as an upper bound on the decrease in profits. It follows that an increase in b must be met with an increase in P otherwise the IFI would earn negative profit and would not

participate in the market. ■

Proof of Lemma 4

Since counterparty risk is defined as $\int_{\underline{R}_f}^{C(\gamma - \beta P^* \gamma)} dF(\theta)$, we are interested in what happens to $C(\gamma - \beta^* P^* \gamma)$ as b changes.

We first focus on the case in which $\beta^* \in (0, 1)$. Defining $\frac{\partial \beta^*}{\partial b} \equiv \frac{\partial \beta}{\partial b} \Big|_{\beta=\beta^*}$ and $\frac{\partial P^*}{\partial b} \equiv \frac{\partial P}{\partial b} \Big|_{P=P^*}$, we take following partial derivative.

$$\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} = -\gamma \left(\frac{\partial \beta^*}{\partial b} P^* + \beta^* \frac{\partial P^*}{\partial b} \right) \quad (25)$$

From Lemma 1 we know $\frac{\partial \beta^*}{\partial b} \geq 0$ and from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Since $\beta^* \in (0, 1)$ and $P^* > 0$ (from Lemma 2), it follows that:

$$\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} < 0 \quad (26)$$

Therefore, as b increases, counterparty risk decreases when $\beta^* \in (0, 1)$. Next, consider the case of $\beta^* = 1$. Again, from Lemma 3 we know $\frac{\partial P^*}{\partial b} > 0$. Therefore, $\frac{\partial (\gamma - \beta^* P^* \gamma)}{\partial b} < 0$ regardless of whether $\frac{\partial \beta^*}{\partial b} = 0$ or $\frac{\partial \beta^*}{\partial b} > 0$. Thus, counterparty risk decreases when b decreases if $\beta^* = 1$.

It is obvious that if $\beta^* = 0$ there will be no change in counterparty risk by noting that $\beta^* P^* \gamma$ will be independent of b . ■

Proof of Proposition 1

We begin by ruling out a separating equilibrium when there is no counterparty risk regardless of the IFI's choice, i.e., $\int_{\underline{R}_f}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta) = 0$. It follows that the left hand side of both (5) and (6) are zero. Since $P_R^* - P_S^* > 0$, (5) and (6) cannot be simultaneously satisfied so that this separating equilibrium cannot exist.

We now focus on the potential pooling equilibria. In such an equilibrium, the IFI does not update its prior so that the perceived probability of a claim is $\frac{1}{2}(2 - p_S - p_R)$. The corresponding investment decision is then $\beta_{1/2}^*$. In the event that an off the equilibrium path message is submitted, the IFI makes an investment decision denoted by β_ψ . There are four possible pooling equilibria.

1. $(SS, \beta_{1/2}^* \beta_{\psi_1})$, 2. $(RR, \beta_{\psi_2} \beta_{1/2}^*)$, 3. $(RR, \beta_{\psi_3} \beta_{1/2}^*)$, 4. $(SS, \beta_{1/2}^* \beta_{\psi_4})$.

The first (second) digit represents the message that a safe (risky) bank sends, while the third (fourth) digit represents the investment decision that the IFI makes upon receiving the safe (risky) message. If it exists, the first equilibrium can be supported by an off equilibrium path belief, $b \in [\frac{2-p_S-p_R}{2}, 1 - p_R]$. $\beta_{\psi_1} \in [\beta_{1/2}^*, \beta_R^*]$ represents the corresponding off the equilibrium path investment decision. For example, if the supporting off the equilibrium path belief is $b = 1 - p_R$,

then the IFI uses β_R^* as its off the equilibrium path investment. The second pooling equilibrium (if it exists) is supported by the same off the equilibrium path beliefs as above, with $\beta_{\psi_2} \in [\beta_{1/2}^*, \beta_R^*]$. The third and fourth pooling equilibria (if they exist) are supported by an off the equilibrium path belief, $b \in [1 - p_S, \frac{2-p_S-p_R}{2}]$, with $\beta_{\psi_3} \in [\beta_S^*, \beta_{1/2}^*]$ and $\beta_{\psi_4} \in [\beta_S^*, \beta_{1/2}^*]$. Notice that the first two pooling equilibria are equivalent in terms of outcome and differ only in how the language (messages) are understood. The understanding of language is not the focus of this paper so we ignore the second pooling equilibrium. The same is the case with the third and fourth pooling equilibrium, so we focus only on the third.

We begin with the first pooling equilibrium. Let $P_{1/2}^*$ (P_{ψ_1}) be the resulting price when the investment decision is $\beta_{1/2}^*$ (β_{ψ_1}). The following two conditions formalize this case:

$$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{\psi_1} P_{\psi_1} \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{\psi_1} - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \quad (27)$$

$$\Pi(R, S) \geq \Pi(R, R) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_{\psi_1} P_{\psi_1} \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)}_{\text{expected cost of the additional counterparty risk}} \leq \underbrace{P_{\psi_1} - P_{1/2}^*}_{\text{amount to be saved in insurance premia}} \quad (28)$$

The binding condition (28) can be satisfied when Z is sufficiently small. Intuitively, this condition is satisfied if the bank wishes only to obtain the lowest premium. In other words, the *premium effect* dominates for both types.

Consider the second potential pooling equilibrium. In this case, let P_{ψ_3} be the resulting price when the investment decision is β_{ψ_3} . The following two conditions formalize this case:

$$\Pi(S, R) \geq \Pi(S, S) \Rightarrow$$

$$\underbrace{(1 - p_S)(1 + Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_{\psi_3} P_{\psi_3} \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2}^* - P_{\psi_3}}_{\text{amount extra to be paid in insurance premia}} \quad (29)$$

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$\underbrace{(1 - p_R)(1 + Z) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_{\psi_3} P_{\psi_3} \gamma)} dF(\theta)}_{\text{expected saving in counterparty risk}} \geq \underbrace{P_{1/2}^* - P_{\psi_3}}_{\text{amount extra to be paid in insurance premia}} \quad (30)$$

The binding condition (29) is satisfied for Z sufficiently large. Intuitively, this condition is satisfied

if the bank is so averse to counterparty risk that the *counterparty risk effect* dominates for both types.

The separating equilibria are given as: $(SR, \beta_S^* \beta_R^*)$ and $(RS, \beta_R^* \beta_S^*)$. As in the case of the pooling equilibrium, we ignore the second separating equilibrium since it leads to the same outcome. Combining (5) and (6), we obtain the following condition for when the separating equilibrium exists.

$$\frac{P_R^* - P_S^*}{(1 - p_R) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \leq 1 + Z \leq \frac{P_R^* - P_S^*}{(1 - p_S) \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{C(\gamma - \beta_S^* P_S^* \gamma)} dF(\theta)} \quad (31)$$

Note that the set defined by (31) is non-empty since $p_S > p_R$. We now return to the pooling equilibria. The condition under which no pooling equilibrium can exist (i.e., when (28) and (29) are not satisfied) can be written as follows.

$$\frac{P_{\psi_1} - P_{1/2}^*}{(1 - p_R) \int_{C(\gamma - \beta_{\psi_1} P_{\psi_1} \gamma)}^{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)} dF(\theta)} < 1 + Z < \frac{P_{1/2}^* - P_{\psi_3}}{(1 - p_S) \int_{C(\gamma - \beta_{1/2}^* P_{1/2}^* \gamma)}^{C(\gamma - \beta_{\psi_3} P_{\psi_3} \gamma)} dF(\theta)} \quad (32)$$

It follows that if (31) and (32) are satisfied, the separating equilibrium exists and is unique for some subset of Z . First consider the case in which β_{ψ_1} and β_{ψ_3} are fixed with $\beta_{\psi_1} > \beta_{1/2}^*$, $\beta_{\psi_3} < \beta_{1/2}^*$ (so that $P_{\psi_1} > P_{1/2}^*$, $P_{\psi_3} < P_{1/2}^*$). To see that (31) and (32) can be simultaneously satisfied, let $p_S \rightarrow 1$ so that the right hand side of both (31) and (32) are satisfied (note that p_R need not approach one so that $P_{1/2}^* > 0$ which in turn implies that the numerator of both expressions converge to a finite number). It follows that if Z is sufficiently large, the left hand side of these two inequalities can be satisfied yielding the desired result.

The above procedure fails if $\beta_{\psi_1} \rightarrow \beta_{1/2}^*$ and/or $\beta_{\psi_3} \rightarrow \beta_{1/2}^*$ since the numerator and denominator of the left and/or right hand side of (32) approach zero. In this case, (32) will never hold, so that a pooling equilibrium always exists. We then need to ask: if separating and pooling equilibria co-exist, can we rule either of them out? We employ the neologism proof criterion proposed by Farrell (1993). Intuitively, this refinement says that if both bank types prefer to reveal truthfully, then there should exist a credible message that relays this information. Note that when $\beta_{\psi_1} = \beta_{\psi_3} = \beta_{1/2}$, this represents the babbling equilibrium in which the IFI's posterior after any message is also its prior. The Farrell (1993) criterion can rule out any pooling equilibrium (not just babbling) that co-exists with separation, provided that separation is preferred by both bank types. Therefore, it can be used to eliminate multiple equilibria when $\beta_{\psi_1} \rightarrow \beta_{1/2}^*$ and/or $\beta_{\psi_3} \rightarrow \beta_{1/2}^*$. The condition under which separation is preferred by both bank types is conveniently given by (32) with $\beta_{\psi_1} = \beta_R^*$ and $\beta_{\psi_3} = \beta_S^*$. Since $\beta_R^* > \beta_{1/2}^*$ and $\beta_S^* < \beta_{1/2}^*$, we showed above that this condition and condition (31) can be simultaneously satisfied. Therefore, there exists a range of Z for which this refinement criterion can eliminate any pooling equilibrium that co-exists with the separating equilibrium. We can now conclude that the separating equilibrium is the unique outcome for some non-empty subset of Z . In terms of other refinements, all three forms of announcement-proofness proposed by Matthews, Okuno-Fujiwara and Postlewaite (1991) select the separating equilibrium

as the unique outcome by the same argument and so work equally well.

The first condition given in the statement of the proposition can be found by re-arranging the right hand sides of (31) and (32) with $\beta_{\psi_1} = \beta_R^*$ and $\beta_{\psi_3} = \beta_S^*$, while the second condition can be found by re-arranging the left hand sides of these inequalities. ■

Proof of Proposition 2

We begin by showing that there is no price $\tilde{P} < P_b^*$ such that the IFI can earn zero profit. It is straight-forward to see that $\Pi_{IFI}(\beta_b^*, P_b^*) = 0$ (where Π_{IFI} is defined by (2)) implies that $\Pi_{IFI}(\tilde{\beta}, \tilde{P}) \neq 0 \forall \tilde{\beta} \in [0, 1]$ and for $\tilde{P} < P_b^*$.

Since Lemmas 1 and 2 show that with (β_b^*, P_b^*) , zero profit is attained, it must be with $\tilde{\beta} \in [0, 1] \neq \beta_b^*$ and P_b^* , the IFI earns negative profits. It follows that with $\tilde{\beta}$ and $\tilde{P} < P_b^*$, the IFI also earns negative profits. Since the IFI must earn zero profits, $\tilde{P} \geq P_b^*$. This implies that $P_b^* \leq P_b^{sp}$.

The proof now proceeds in 3 steps. Step 1 derives the first order condition for the planning problem. Step 2 assumes the equilibrium solution and derives an expression for $\frac{\partial P}{\partial \beta}$ from the IFI's zero profit condition. Step 3 shows that β^{sp} and P^{sp} must be greater than in the equilibrium case when $\beta^* < 1$. Since we need not specify a belief for this proof, it follows that the result holds regardless of whether there is separation or pooling of banks.

Step 1

The result is valid for both the ex-ante case and the case in which the types are known. We use the case when the types are known; however, after the first order condition (33), we state a simple redefinition of the default probability parameter that will yield the ex-ante case. The profit for the bank (bk) of type $j \in \{S, R\}$ can be written as follows.

$$\Pi_{bk} = p_j R_B \gamma + \gamma(1 - p_j) \int_{C(\gamma - \beta P \gamma)}^{\bar{R}_f} dF(\theta) - \gamma(1 - p_j) Z \int_{\underline{R}_f}^{C(\gamma - \beta P \gamma)} dF(\theta) - \gamma P$$

In the planners case, P^{sp} is now endogenous and determined by $\Pi_{IFI}(\beta^{sp}, P^{sp}) = 0$ (where Π_{IFI} is defined by (2)). Using the uniform assumption on F yields the following first order condition.

$$\frac{\partial P}{\partial \beta} = \gamma C'(\gamma - \beta P \gamma) \left(P + \frac{\partial P}{\partial \beta} \beta \right) (1 - p_j)(1 + Z) \quad (33)$$

The left hand side represents the marginal cost of increasing β , while the right hand side represents the marginal benefit of doing so. Note that if we had derived this expression using the expected profit, then $(1 - p_j) = \frac{1}{2}(2 - p_R - p_S)$.

Step 2

We show that if $\beta^{sp} = \beta^*$, then (33) cannot hold. We find an expression for $\frac{\partial P}{\partial \beta} \Big|_{\beta=\beta^*, P=P^*}$ by

implicitly differentiating the equation $\Pi_{IFI}(\beta^*, P^*) = 0$.

$$0 = (1-b) \left[\int_{-P^*\gamma(\beta^*+(1-\beta^*)R_I)}^{\bar{R}_f} (\theta + P^*\gamma(\beta^* + (1-\beta^*)R_I)) dF(\theta) \right] \\ + (b) \left[\int_{C(\gamma-\beta^*P^*\gamma)}^{\bar{R}_f} (\theta - C(\gamma - \beta^*P^*\gamma) + P^*\gamma(1-\beta^*)R_I) dF(\theta) \right]$$

We implicitly differentiate this equation (assuming $f(\cdot)$ is normally distributed) to find $\frac{\partial P}{\partial \beta} \Big|_{\beta=\beta^*, P=P^*}$.

$$A_2 \frac{\partial P}{\partial \beta} \Big|_{\beta=\beta^*, P=P^*} = -\frac{bP^*\gamma}{\bar{R}_f - \underline{R}_f} \left[\bar{R}_f C'(\gamma - \beta^*P^*\gamma) - C'(\gamma - \beta^*P^*\gamma)C(\gamma - \beta^*P^*\gamma) \right. \\ \left. - \bar{R}_f R_I + C'(\gamma - \beta^*P^*\gamma)P^*\gamma(1-\beta^*)R_I + C(\gamma - \beta^*P^*\gamma)R_I \right] \\ + (1-b) \frac{P^*\gamma}{\bar{R}_f - \underline{R}_f} (R_I - 1) [\bar{R}_f + (\beta^* + (1-\beta^*)R_I)] \quad (34)$$

Where we define:

$$A_2 = \frac{b}{\bar{R}_f - \underline{R}_f} \left[-\bar{R}_f C'(\gamma - \beta^*P^*\gamma)\beta\gamma + \bar{R}_f(1-\beta^*)R_I\gamma \right. \\ \left. - C'(\gamma - \beta^*P^*\gamma)C(\gamma - \beta^*P^*\gamma)\beta^*\gamma + C'(\gamma - \beta^*P^*\gamma)\beta^*\gamma(1-\beta^*)R_I P^*\gamma \right. \\ \left. - C(\gamma - \beta^*P^*\gamma)(1-\beta^*)R_I\gamma \right] \\ + \frac{(1-b)}{\bar{R}_f - \underline{R}_f} [\gamma(\beta^* + (1-\beta^*)R_I) (\bar{R}_f + P^*\gamma(\beta^* + (1-\beta^*)R_I))] . \quad (35)$$

It follows that $\frac{\partial P}{\partial \beta} \Big|_{\beta=\beta^*, P=P^*} = 0$ since the right hand side of (34) is the FOC derived in Lemma 1 and must equate to 0 at the optimum, β^* .

Step 3

Substituting $\frac{\partial P}{\partial \beta} \Big|_{\beta=\beta^*, P=P^*} = 0$ into (33) yields:

$$0 = \gamma C'(\gamma - \beta^*P^*\gamma) (P^*) (1 - p_j)(1 + Z), \quad (36)$$

which cannot hold since $\gamma > 0$, $(1 - p_j) > 0$ and $Z > 0$. Therefore, $\beta^{sp} \neq \beta^*$ and $P^{sp} \neq P^*$. To satisfy (33), it must be the case that $\beta^{sp} > \beta^*$. Since β^* was profit maximizing for the IFI, and with (β^*, P^*) the IFI earned zero profit, it follows that profit must be negative with (β^{sp}, P^*) . Therefore, $P^{sp} > P^*$ must hold so that the IFI earns zero profit when its investment choice is β^{sp} .

This implies that the following must hold.

$$\int_0^{C(\gamma - \beta^{sp} P^{sp} \gamma)} f(\theta) d\theta < \int_0^{C(\gamma - \beta^* P^* \gamma)} f(\theta) d\theta \quad (37)$$

Therefore, there is strictly less counterparty risk in the planner's case than in the equilibrium case. It is obvious that if $\beta^* = 1$ (the IFI invests everything in the liquid asset), the planner sets $\beta^{sp} = 1$ and the counterparty risk does not change. ■

Proof of Lemma 5

Our problem can be written as follows (recalling that $f(\cdot)$ is assumed to be uniformly distributed).

$$\begin{aligned} \max_{\beta} \quad & \frac{1}{\bar{R}_f - \underline{R}_f} \left\{ \int_0^{\beta PM} \left[\frac{(\bar{R}_f)^2}{2} - \frac{(PM\gamma(\beta + (1 - \beta)R_I) - y\gamma)^2}{2} \right] db(y) \right. \\ & + \int_{\beta PM}^M \left[\frac{(\bar{R}_f)^2}{2} - \frac{(C(y\gamma - \beta PM\gamma))^2}{2} \right] db(y) \\ & + \int_0^{\beta PM} (\bar{R}_f + (PM\gamma(\beta + (1 - \beta)R_I) - y\gamma)) (PM\gamma(\beta + (1 - \beta)R_I) - y\gamma) db(y) \\ & \left. + \int_{\beta PM}^M (\bar{R}_f - C(y\gamma - \beta PM\gamma)) (-C(y\gamma - \beta PM\gamma) + (1 - \beta)R_I PM\gamma) db(y) \right\} \end{aligned}$$

Taking the FOC yields the following.

$$\begin{aligned} 0 = \quad & \frac{PM\gamma}{\bar{R}_f - \underline{R}_f} \int_0^{\beta^* PM} [-\bar{R}_f(R_I - 1) - (PM\gamma(\beta^* + (1 - \beta^*)R_I) - y\gamma)(R_I - 1)] db(y) \\ & + \frac{PM}{\bar{R}_f - \underline{R}_f} \left[\frac{(\bar{R}_f)^2}{2} + \bar{R}_f(1 - \beta^*)R_I PM\gamma + \frac{1}{2} (PM\gamma(1 - \beta^*)R_I)^2 \right] \\ & + \frac{PM\gamma}{\bar{R}_f - \underline{R}_f} \int_{\beta^* PM}^M [C'(y\gamma - \beta^* PM\gamma) (\bar{R}_f - C(y\gamma - \beta^* PM\gamma)) - \bar{R}_f R_I] db(y) \\ & + \frac{PM\gamma}{\bar{R}_f - \underline{R}_f} \int_{\beta^* PM}^M [C'(y\gamma - \beta^* PM\gamma)(1 - \beta^*)R_I PM\gamma + C(y\gamma - \beta^* PM\gamma)R_I] db(y) \\ & - \frac{PM}{\bar{R}_f - \underline{R}_f} \left[\frac{(\bar{R}_f)^2}{2} - \frac{(C(0))^2}{2} + (\bar{R}_f - C(0)) (-C(0) + (1 - \beta^*)R_I PM\gamma) \right] \quad (38) \end{aligned}$$

Recalling $C(0) = 0$ we simplify the above.

$$\begin{aligned}
0 &= - \int_0^{\beta^* PM} [\bar{R}_f(R_I - 1) + (PM\gamma(\beta^* + (1 - \beta^*)R_I) - y\gamma)(R_I - 1)] db(y) \\
&+ \int_{\beta^* PM}^M [(C'(y\gamma - \beta^* PM\gamma) - R_I) (\bar{R}_f - C(y\gamma - \beta^* PM\gamma))] db(y) \\
&+ \int_{\beta^* PM}^M [C'(y\gamma - \beta^* PM\gamma)(1 - \beta^*)PM\gamma] db(y) \\
&+ \frac{1}{2} (PM\gamma(1 - \beta^*)R_I)^2
\end{aligned} \tag{39}$$

The SOC (which we omit for brevity) implies that the right hand side of (39) is decreasing in β^* so that our problem achieves a maximum. Define two belief distributions $b_1(y)$ and $b_2(y)$ such that $b_1(y) \geq b_2(y) \forall y$. As well, let $(\beta_1^*, b_1(y))$ solve the first order condition (39). Intuitively, by moving from $b_1(y)$ to $b_2(y)$, mass shifts from the interval $[0, \beta^* PM]$ to $[\beta^* PM, M]$. Formally:

$$\int_0^{\beta^* PM} db_1(y) > \int_0^{\beta^* PM} db_2(y) \tag{40}$$

$$\int_{\beta^* PM}^M db_1(y) < \int_{\beta^* PM}^M db_2(y). \tag{41}$$

We wish to show that by moving from $b_1(y)$ to $b_2(y)$, β^* increases. We do this in two cases that span the parameter space and show that the result is the same in both. First consider the case in which $C'(y\gamma - \beta^* PM\gamma) \geq R_I$. In this case, the first term on the right hand side of (39) is negative while the second two are positive. This implies that as mass shifts from the first term to the second and third terms, the right hand side of (39) increases. Since this equation is decreasing in β^* (by the SOC), β^* must increase as desired.

Next, consider the case in which $C'(y\gamma - \beta^* PM\gamma) \leq R_I$. Rearranging the FOC (39) we obtain the following.

$$\begin{aligned}
0 &= - \int_0^{\beta^* PM} [(PM\gamma(\beta^* + (1 - \beta^*)R_I) - y\gamma)(R_I - 1)] db(y) \\
&+ \int_{\beta^* PM}^M [\bar{R}_f (C'(y\gamma - \beta^* PM\gamma) - 1)] db(y) \\
&+ \int_{\beta^* PM}^M [C(y\gamma - \beta^* PM\gamma) (R_I - C'(y\gamma - \beta^* PM\gamma))] db(y) \\
&+ \int_{\beta^* PM}^M [C'(y\gamma - \beta^* PM\gamma)(1 - \beta^*)PM\gamma] db(y) \\
&+ \frac{1}{2} (PM\gamma(1 - \beta^*)R_I)^2 - \bar{R}_f(R_I - 1)
\end{aligned} \tag{42}$$

It follows that the first term on the right hand side of (42) is trivially negative, while the second term is positive since $C'(y\gamma - \beta^* PM\gamma) > 1$ (recall that $C(x) \geq x$ was assumed so that $C'(x) \geq 1$)

and $R_I \geq C'(y\gamma - \beta^*PM\gamma)$. The third term is trivially positive. The results from the first case then hold here, namely, a change from $b_1(y)$ to $b_2(y)$ implies that β^* increases. It follows that the riskier the beliefs about the distribution of loans that the IFI insures, the more the IFI invests in the liquid asset.

To proceed, we use a similar result to that of Lemma 3. It is straight forward to see that when the belief of defaults is higher (as in the risky case), so must the price of the contracts be higher (this can be proved in the same way as Lemma 3 by showing that the profit function is decreasing in the amount of risk in the loans). Next we find what happens to counterparty risk. What is different about the case of multiple banks is that counterparty risk is defined relative to the number of banks that default: $\int_{\beta_{PM}}^M \int_{R_f}^{C(y\gamma - \beta PM\gamma)} dF(\theta) db(y)$.

In the case in which the IFI puts more weight on the loans being risky ($q_A = r$), we showed that β^* and P^* increase, so that $C(\gamma - \beta P\gamma)$ decreases. Since, from the point of view of the banks the probability of a claim does not change, counterparty risk decreases as compared to the case in which the IFI puts more weight on the loans being safe ($q_A = s$).

■

Proof of Proposition 3

The proof proceeds in 3 steps and follows in the same spirit as the proof to Proposition 1. In the first step, we show when a pooling equilibrium cannot exist. In the second step we show when a separating equilibrium must exist. In the final step we show that there exists a parameter space in which separation is the unique outcome. As in the proof Proposition 1, we ignore the issue of how language (messages) are understood. For example, we assume that the IFI interprets the message $q_A = s$ as a statement that the type is safe. If on the other hand, the interpretation of the message $q_A = s$ is that the bank is stating that it is risky, then a safe bank must send the message $q_A = r$ to convey that it is safe. This issue is not of interest here since it leads to the same outcome as the case in which the signals are sent and understood correctly.

We assume that the IFI processes the reports of all banks and forms its beliefs based on the proportion that submit the report $q_A = s$ and $q_A = r$. Define $\#s$ to be the number of banks that report $q_A = s$. The number of banks that report $q_A = r$ is then given by $M - \#s$. In what follows, define a bank as *pivotal* if by changing its report, it can change the beliefs of the IFI.

Step 1

Assume that the IFI does not update its prior if $\#s \geq \alpha_1$ where $\alpha_1 \in [0, M]$. The corresponding investment decision is $\beta_{1/2}^*$ with price $P_{1/2}^*$. If $\#s < \alpha_1$, then the IFI has an off the equilibrium path investment decision $\beta_{\psi_1} \in [\beta_{1/2}^*, \beta_r^*]$ (with corresponding price P_{ψ_1}) supported by off the equilibrium path beliefs that $q_A \in [\frac{s+r}{2}, r]$. For example, if $\beta_{\psi_1} = \beta_r^*$ then the supporting off the equilibrium path belief is $q_A = r$. To determine when pooling (in outcomes)³⁴ cannot exist, we use beliefs about

³⁴By “pooling (in outcomes)”, we mean that the IFI does not update its beliefs, even in cases where some banks

the idiosyncratic shock which make deviation the most difficult to support. In particular, assume that a deviating bank pays $P_{\psi_1}^M$, while the IFI still solves its problem with P_{ψ_1} as the “average” price.³⁵ In other words, a deviating bank is assumed to have received an idiosyncratic shock of M , the worst possible draw.³⁶ We now find a condition under which deviation is profitable. Since a bank has a stronger incentive to deviate if $q_A = r$ than if $q_A = s$, it suffices to check the former. Furthermore, the lower the idiosyncratic shock, the less is the incentive to deviate. Therefore, we consider the bank that received the lowest idiosyncratic shock, q_r^0 (which represents its probability of default). Next, let $D_{1/2}$ (D_{ψ_1}) represent the probability that upon a claim being made in the pooling (deviating) case, the IFI fails and so cannot pay. These variables are given as follows.

$$D_{1/2} = \int_{\beta_{1/2}^* P_{1/2}^* M}^M \int_{\underline{R}_f}^{C(y\gamma - \beta_{1/2}^* P_{1/2}^* M\gamma)} dF(\theta) db(y)$$

$$D_{\psi_1} = \int_{\beta_{\psi_1} P_{\psi_1} M}^M \int_{\underline{R}_f}^{C(y\gamma - \beta_{\psi_1} P_{\psi_1} M\gamma)} dF(\theta) db(y)$$

We can now give the condition under which this pooling equilibrium exists (note that in this case and the cases to follow, the payoff to the bank if there is no claim does not affect the bank's choice and so it is dropped).

$$q_r^0 \gamma (1 - D_{\psi_1}) - q_r^0 \gamma D_{\psi_1} Z - \gamma P_{\psi_1}^M \leq q_r^0 \gamma (1 - D_{1/2}) - q_r^0 \gamma D_{1/2} Z - \gamma P_{1/2}^*$$

$$\Rightarrow q_r^0 (D_{1/2} - D_{\psi_1}) (1 + Z) \leq P_{\psi_1}^M - P_{1/2}^* \quad (43)$$

Now consider a different set of beliefs for the IFI. Assume that the IFI does not update its prior if $\#s \leq \alpha_2$ where $\alpha_2 \in [0, M]$. The corresponding investment decision is $\beta_{1/2}^*$ with price $P_{1/2}^*$. If $\#s > \alpha_2$, then the IFI has an off the equilibrium path investment decision $\beta_{\psi_2} \in [\beta_s^*, \beta_{1/2}^*]$ (with corresponding price P_{ψ_2}) supported by off the equilibrium path beliefs that $q_A \in [s, \frac{s+r}{2}]$. We define D_{ψ_2} in a similar way as D_{ψ_1} and $D_{1/2}$ given above. We assume that the beliefs are that a deviating bank received the worst idiosyncratic shock, M and consider the case in which $q_A = s$.

submit different reports. For the remainder of the proof, we will refer to this as a pooling equilibrium. Similarly, we will refer to separation in outcomes as a separating equilibrium.

³⁵There is a technical issue that would not arise if there was a finite number of banks. In the traditional Riemann sense of measurability using the concept of point-wise convergence almost everywhere, a bank of measure zero cannot change the IFI's beliefs. There are three ways to rectify this. The first way is to think of the number of banks as finite but large, so that the continuous case is used as an approximation. The second way is to employ the Pettis-integral as in Uhlig (1996). The final way is to imagine a small but positive measure of banks deviating.

³⁶There is a version of a free-riding problem referred to as a volunteering problem that must be considered. If it is profitable for a bank to deviate, then it would be more profitable for it if another bank deviates. The reason for this is that the deviating bank is believed to have received the worst idiosyncratic shock. In the case we explore, we find a condition under which it would be individually profitable for any bank to deviate, given that no other bank was deviating. Although we cannot tell which bank would actually deviate in equilibrium, this does not hamper our ability to rule out these pooling equilibria.

The condition under which this pooling equilibrium exists is given as follows.

$$\begin{aligned} q_s^M \gamma(1 - D_{\psi_2}) - q_s^M \gamma D_{\psi_2} Z - \gamma P_{\psi_2}^M &\leq q_s^M \gamma(1 - D_{1/2}) - q_s^M \gamma D_{1/2} Z - \gamma P_{1/2}^* \\ &\Rightarrow q_s^M (D_{\psi_2} - D_{1/2}) (1 + Z) \geq P_{1/2}^* - P_{\psi_2}^M \end{aligned} \quad (44)$$

Therefore, pooling cannot occur whenever both (43) and (44) are simultaneously not met. This condition is given as follows.

$$\frac{P_{\psi_1}^M - P_{1/2}^*}{q_r^0 (D_{1/2} - D_{\psi_1})} < 1 + Z < \frac{P_{1/2}^* - P_{\psi_2}^M}{q_s^M (D_{\psi_2} - D_{1/2})} \quad (45)$$

Step 2

We now turn to the possibility of separation. Let the beliefs of the IFI be that if $\#s \geq \alpha_3$ ($\#s < \alpha_4$) then the investment decision is β_s^* (β_r^*), where $\alpha_3 \in [0, M]$, $\alpha_4 \in (0, M]$, and $\alpha_3 \geq \alpha_4$. If $\alpha_3 > \alpha_4$ and $\alpha_4 \leq \#s < \alpha_3$, then the investment decision is off the equilibrium path and chosen from the open set (β_s^*, β_r^*) , supported by beliefs that $q_A \in (s, r)$. Note that when $\alpha_3 = \alpha_4$, the IFI's beliefs are fully defined by Bayes' rule and so no off equilibrium path beliefs are required.

Consider the case in which the true state is $q_A = s$ and $\#s = \alpha_3$. We find the condition under which there is no (pivotal) bank that wishes to deviate. Assume that the investment decision of the IFI in the event of a deviation is $\beta_{\psi_3} \in (\beta_s^*, \beta_r^*)$ so that counterparty risk is given by D_{ψ_3} . We assume that a pivotal deviating bank pays the premium corresponding to the lowest idiosyncratic shock $P_{\psi_3}^0$. We use the bank that could benefit the most from deviation, q_s^M . Therefore, if this bank does not deviate, then no other (safe aggregate type) bank would. Define D_s and D_{ψ_3} in the usual way so that the condition under which this bank would report truthfully is given by the following.

$$\begin{aligned} q_s^M \gamma(1 - D_s) - q_s^M \gamma D_s Z - \gamma P_s^* &\geq q_s^M \gamma(1 - D_{\psi_3}) - q_s^M \gamma D_{\psi_3} Z - \gamma P_{\psi_3}^0 \\ &\Rightarrow P_{\psi_3}^0 - P_s^* \geq q_s^M (D_s - D_{\psi_3}) (1 + Z) \end{aligned} \quad (46)$$

Next, consider the case in which the true state is $q_A = r$ and $\#s = \alpha_4 - \epsilon$ where ϵ is arbitrarily close to zero (so that a pivotal bank can change the beliefs of the IFI, i.e., if a bank changes its report from $q_A = r$ to $q_A = s$, then $\#s = \alpha_4$ which is off the equilibrium path). Assume that the investment decision of the IFI in the event of a deviation is β_{ψ_4} so that counterparty risk is given by D_{ψ_4} . We assume that a pivotal deviating bank pays the premium $P_{\psi_4}^0$ and consider the bank q_r^0 . Define D_r and D_{ψ_4} in the usual way so that the condition under which this bank would report truthfully is given by the following.

$$\begin{aligned} q_r^0 \gamma(1 - D_r) - q_r^0 \gamma D_r Z - \gamma P_r^* &\geq q_r^0 \gamma(1 - D_{\psi_4}) - q_r^0 \gamma D_{\psi_4} Z - \gamma P_{\psi_4}^0 \\ &\Rightarrow q_r^0 (D_{\psi_4} - D_r) (1 + Z) \geq P_r^* - P_{\psi_4}^0 \end{aligned} \quad (47)$$

Inequalities (46) and (47) are simultaneously satisfied when:

$$\frac{P_r^* - P_{\psi_4}^0}{q_r^0 (D_{\psi_4} - D_r)} \leq 1 + Z \leq \frac{P_{\psi_3}^0 - P_s^*}{q_s^M (D_s - D_{\psi_3})}. \quad (48)$$

Step 3

It follows that if (45) and (48) are satisfied, the separating equilibrium exists and is unique for some subset of Z . Let $M \rightarrow 0$ so that $P_{\psi_1}^M \rightarrow P_{\psi_1}$, $P_{\psi_2}^M \rightarrow P_{\psi_2}$, $P_{\psi_3}^0 \rightarrow P_{\psi_3}$, $P_{\psi_4}^0 \rightarrow P_{\psi_4}$, and let $s = 0$ so that $q_s^M \rightarrow 0$. Consider the case in which the off the equilibrium path beliefs are such that $\beta_{\psi_3} \rightarrow \beta_s^*$ (so that $P_{\psi_3} \rightarrow P_s^*$ and $D_{\psi_3} \rightarrow D_s$). In this case, (46) is trivially satisfied so that no safe bank wishes to deviate from the separating equilibrium. If $\beta_{\psi_4} \rightarrow \beta_r^*$ (so that $P_{\psi_4} \rightarrow P_r^*$ and $D_{\psi_4} \rightarrow D_r$), (47) is trivially satisfied and so no risky bank wishes to deviate from the separating equilibrium. Therefore, we consider the case in which $D_{\psi_3} < D_s$, $P_{\psi_3} > P_s^*$ and $D_{\psi_4} > D_r$, $P_{\psi_4} < P_r^*$.

Let β_{ψ_1} and β_{ψ_2} be fixed with $\beta_{\psi_1} > \beta_{1/2}^*$ (so that $P_{\psi_1} > P_{1/2}^*$ and $D_{\psi_1} < D_{1/2}$) and $\beta_{\psi_2} < \beta_{1/2}^*$ (so that $P_{\psi_2} < P_{1/2}^*$ and $D_{\psi_2} > D_s$). Since $q_s^M \rightarrow 0$, $P_{1/2}^* > P_{\psi_2}$ and $P_{\psi_3} > P_s^*$, the right hand side of both (45) and (48) become infinite and so are satisfied for Z arbitrarily large. If we assume $r > s + M$ (which implies $q_s^M < q_r^0$ so that we can choose $q_r^0 > 0$), then there exists a finite (range of) Z which satisfies the right hand side of both (45) and (48) and also satisfies the left hand side of these conditions. Note that if we do not assume $r > s + M$, then existence of a separating equilibrium can be established simply by choosing off the equilibrium path beliefs such that (48) is satisfied.

The above procedure fails if $\beta_{\psi_1} \rightarrow \beta_{1/2}^*$ and/or $\beta_{\psi_2} \rightarrow \beta_{1/2}^*$ since the numerator and denominator of the left and/or right hand side of (45) approach zero. We take the same approach as in the proof to Proposition 1, and employ the neologism proof criteria proposed by Farrell (1993). We need to show that for some parameter range in which separation and pooling co-exist, pooling can be eliminated. Note that the Farrell (1993) criterion can rule out all pooling (not just babbling) equilibria that co-exist with the separating equilibrium provided that separation is preferred by both bank types. Therefore, it can be used to eliminate multiple equilibria when $\beta_{\psi_1} \rightarrow \beta_{1/2}^*$ and/or $\beta_{\psi_2} \rightarrow \beta_{1/2}^*$. The condition under which separation is preferred by both bank types is conveniently given by (45) with $\beta_{\psi_1} = \beta_r^*$ and $\beta_{\psi_2} = \beta_s^*$. Since $\beta_r^* > \beta_{1/2}^*$ and $\beta_s^* < \beta_{1/2}^*$, we showed above that this condition and condition (48) can simultaneously hold. Therefore, there exists a range of Z for which this refinement criterion can eliminate any pooling equilibrium that co-exists with the separating equilibrium. We can now conclude that the separating equilibrium is the unique outcome for some non-empty subset of Z . In terms of other refinements, all three versions of the announcement proof criterion proposed by Matthews, Okuno-Fujiwara and Postlewaite (1991) select the separating equilibrium as the unique outcome by the same argument and so work equally well. ■

VII.B. APPENDIX B

Formal Analysis of Section III.C.

To ease exposition, define the following variables.

$$A_S^{sp} \equiv \int_{C(\gamma - \beta_S^{sp} P_S^{sp} \gamma)}^{\bar{R}_f} dF(\theta) \quad , \quad A_S^* \equiv \int_{C(\gamma - \beta_S^* P_S^* \gamma)}^{\bar{R}_f} dF(\theta)$$

$$A_R^{sp} \equiv \int_{C(\gamma - \beta_R^{sp} P_R^{sp} \gamma)}^{\bar{R}_f} dF(\theta) \quad , \quad A_R^* \equiv \int_{C(\gamma - \beta_R^* P_R^* \gamma)}^{\bar{R}_f} dF(\theta)$$

We now define the two conditions needed to support the separating equilibrium in the traditional mechanism. First, the safe type must wish to reduce its coverage.

$$\Pi(S, S) \geq \Pi(S, R) \Rightarrow$$

$$p_S R_B + \phi \gamma (1 - p_S) A_S^{sp} - (1 - \phi) \gamma (1 - p_S) Z A_S^{sp} - \gamma (1 - p_S) Z (1 - A_S^{sp}) - \phi \gamma P_S^{sp, \phi} \\ \geq p_S R_B + \gamma (1 - p_S) A_S^{sp} - \gamma (1 - p_S) Z (1 - A_S^{sp}) - \gamma P_R^{sp} \quad (49)$$

The second term on the first line represents the payoff to the bank when the loan fails and the insurer is solvent (which occurs with probability A_S^{sp}). Notice that the bank receives only a proportion $\phi \gamma$ owing to the reduction in coverage. The third term represents the penalty on the bank for reducing its coverage when the insurer succeeds. The fourth term represents the penalty on the bank when the insurer fails, which occurs probability $1 - A_S^{sp}$ (which represents the counterparty risk that the safe type incurs when there is no moral hazard). The final term represents the insurance premium. The second line represents the payoff to the safe type if it does not reduce its coverage, i.e., chooses $\phi = 1$. We use the superscript (sp, ϕ) on the premium to differentiate it from P_S^{sp} (the case in which $\phi = 1$). We now find the condition under which the risky bank does not imitate the safe bank.

$$\Pi(R, R) \geq \Pi(R, S) \Rightarrow$$

$$p_R R_B + \gamma (1 - p_R) A_R^{sp} - \gamma (1 - p_R) Z (1 - A_R^{sp}) - \gamma P_R^{sp} \\ \geq p_R R_B + \phi \gamma (1 - p_R) A_S^{sp} - (1 - \phi) \gamma (1 - p_R) Z A_S^{sp} - \gamma (1 - p_R) Z (1 - A_S^{sp}) - \phi \gamma P_S^{sp, \phi} \quad (50)$$

The condition under which (49) and (50) are satisfied and the separating equilibrium prevails is given by:

$$\frac{(1 - p_R) A_R^{sp} (1 + Z) - P_R^{sp}}{(1 - p_R) A_S^{sp} (1 + Z) - P_S^{sp, \phi}} \geq \phi \geq \frac{(1 - p_S) A_R^{sp} (1 + Z) - P_R^{sp}}{(1 - p_S) A_S^{sp} (1 + Z) - P_S^{sp, \phi}} \quad (51)$$

or,

$$\frac{P_R^{sp} - \phi P_S^{sp,\phi}}{(A_R^{sp} - \phi A_S^{sp})(1 - p_R)} \leq 1 + Z \leq \frac{P_R^{sp} - \phi P_S^{sp,\phi}}{(A_R^{sp} - \phi A_S^{sp})(1 - p_S)}. \quad (52)$$

Condition (51) is standard; the safe bank must reduce its coverage sufficiently so that the risky bank does not imitate, but the reduction must be profitable. We are interested in the case in which, all else equal, the safe bank prefers more coverage. This condition is given by $\Pi_\phi(S, S) > 0$ as follows.

$$(1 - p_S)(1 + Z)A_S^{sp} - \frac{\partial(\phi P_S^{sp,\phi})}{\partial\phi} > 0 \quad (53)$$

Since the safe bank maximizes its coverage (condition on separation being achieved), (52) implies that it chooses ϕ such that:

$$\frac{P_R^{sp} - \phi P_S^{sp,\phi}}{(A_R^{sp} - \phi A_S^{sp})(1 - p_R)} = 1 + Z. \quad (54)$$

By construction, the safe and risky bank suffer different levels of counterparty risk when they are revealed in the traditional separation mechanism (recall that $A_S^{sp} > A_R^{sp}$). We are interested in the case in which this difference is insufficient to induce separation when $\phi = 1$. In other words, we analyze the case in which the safe bank must reduce its coverage to separate in the traditional separation mechanism. The following assumption ensures that if the safe bank does not reduce its coverage (i.e., sets $\phi = 1$), then the risky bank will have the incentive to imitate the safe bank and report that it is safe.

ASSUMPTION 1 *Assume $(1 + Z)(1 - p_R)(A_R^{sp} - A_S^{sp}) < P_R^{sp} - P_S^{sp}$ so that (52) is violated and separation cannot be achieved.*

To investigate the relative welfare costs, we assume that the price in the counterparty risk case can be directly compared with the price in the traditional mechanism. In particular, by revealing itself in the counterparty risk case, the safe bank receives (expected) coverage equal to A_S^* . We assume that if the safe bank reduces its coverage in the traditional separating case to the same amount (i.e., $\phi A_S^{sp} = A_S^*$), it will pay the same premium (i.e., $\phi P_S^{sp,\phi} = P_S^*$). The following assumption formalizes.

ASSUMPTION 2 *If $\phi A_S^{sp} = A_S^*$ then $\phi P_S^{sp,\phi} = P_S^*$.*

We can now investigate the relative welfare costs. Let W_{sep}^{tr} denote the welfare with the traditional separation mechanism, while the welfare with the counterparty risk mechanism is represented as W_{sep} as in Section III.B. Finally, let the difference in welfare costs be represented by \mathcal{W}_2 which is

given by the following.

$$\begin{aligned}
\mathcal{W}_2 &= W_{sep}^{tr} - W_{sep} \\
&= \frac{1}{2}\gamma \left[(1 - p_R)(1 + Z)(A_R^{sp} - A_R^*) - (P_R^{sp} - P_R^*) \right. \\
&\quad \left. + (1 - p_S)(1 + Z)(\phi A_S^{sp} - A_S^*) - (\phi P_S^{sp,\phi} - P_S^*) \right] \tag{55}
\end{aligned}$$

The following proposition determines that the counterparty risk mechanism is at least as costly as the traditional separation mechanism, i.e., $\mathcal{W}_2 \geq 0$. The intuition for this result is given in section III.C.

PROPOSITION 4 *The counterparty risk separation mechanism is at least as costly as the traditional separation mechanism.*

Proof.

From Proposition 2, we know that $A_R^{sp} \geq A_R^*$. Consider the case in which $A_R^{sp} = A_R^*$ so that $P_R^* = P_R^{sp}$. This is the case in which there is no moral hazard problem for the risky bank. It follows that:

$$\mathcal{W}_2 = \frac{1}{2}\gamma \left[(1 - p_S)(1 + Z)(\phi A_S^{sp} - A_S^*) - (\phi P_S^{sp,\phi} - P_S^*) \right]. \tag{56}$$

Modifying (31) to include the definition of A_R^* and A_S^* , it follows that the risky bank reports truthfully in the counterparty risk separation case when the following holds.

$$\frac{P_R^* - P_S^*}{(A_R^* - A_S^*)(1 - p_R)} \leq 1 + Z \tag{57}$$

Consider the case in which (57) holds with equality. It follows from condition (54) that the safe bank chooses ϕ_1 such that $\phi_1 A_S^{sp} = A_S^*$. In other words, the reduction in (expected) coverage that the safe type receives in the counterparty risk separation case is just enough to induce separation. The safe bank in the traditional separation case then must chose the same reduction in coverage to induce separation. Assumption 2 then implies $\phi P_S^{sp,\phi} = P_S^*$ so that $\mathcal{W}_2 = 0$. If (57) holds with strict inequality then (54) implies that the safe bank chooses $\phi_2 > \phi_1$ such that $\phi_2 A_S^{sp} > A_S^*$. This is the case in which the freedom to choose the level of coverage benefits the safe bank in the traditional separation case. The safe bank receives more protection in the traditional separating case while still being revealed as safe. Since $\phi_2 > \phi_1$, condition (53) (which ensures that the safe bank prefers more protection) implies that $\mathcal{W}_2 > 0$ so that the traditional mechanism is unambiguously less costly.

Now consider the case in which $A_R^{sp} > A_R^*$ so that \mathcal{W}_2 is represented by (55) again. Proposition 2 implies that $(1 - p_R)(1 + Z)(A_R^{sp} - A_R^*) - (P_R^{sp} - P_R^*) \geq 0$. Regardless of whether (57) holds with equality or strict inequality, since $A_R^{sp} > A_R^*$, condition (54) implies that $\phi A_S^{sp} > A_S^*$. A similar argument as above then yields $\mathcal{W}_2 > 0$ in this case. Therefore, $\mathcal{W}_2 \geq 0$ so that the counterparty risk separation mechanism is at least as costly as the traditional separation mechanism.

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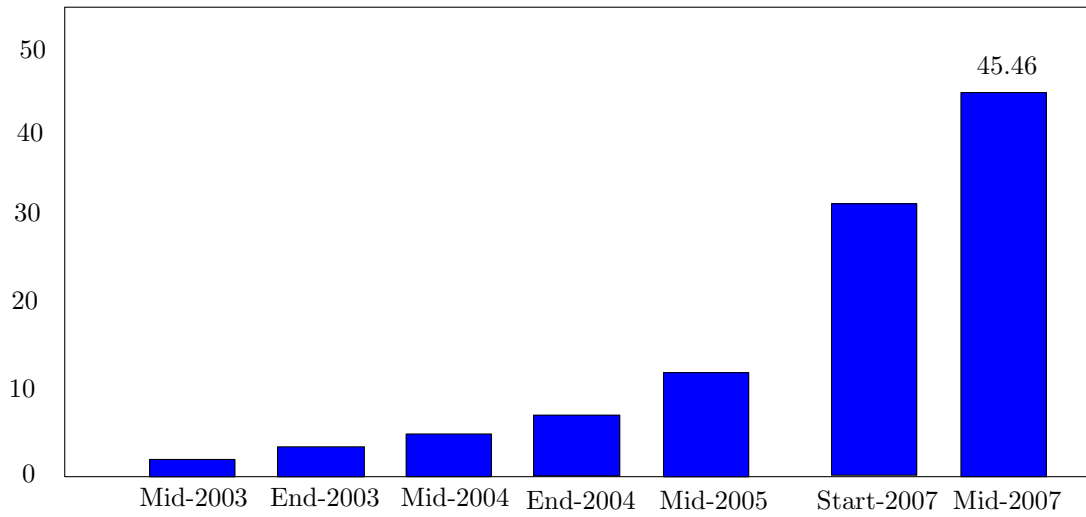


Figure I: Notional Value of Credit Derivatives (in Trillions of Dollars)³⁷

³⁷Source: International Swaps and Derivatives Association Operations Benchmark Survey and FpML Survey (2005,2007). Note that notional value can be a bit misleading when trying to gauge true risk transfer. Notional value represents the amount that must be paid if there is no recovery value.

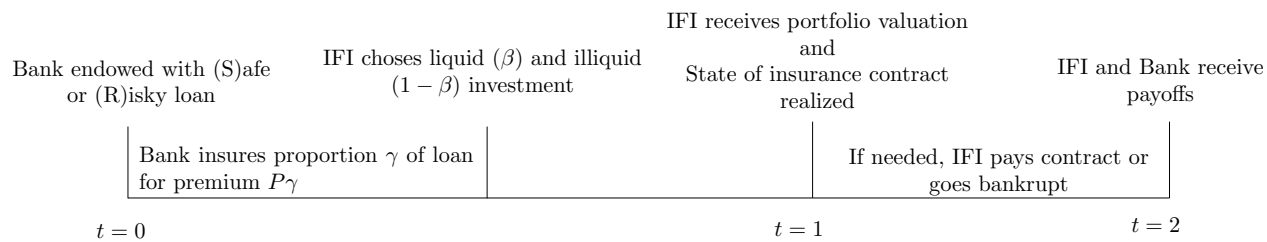


Figure II: Timing of the Model