Modelling Leverage Effect with Copulas and Realized Volatility

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Abstract

In this paper, we propose the use of the static and dynamic copulas to study so-called leverage effect in equity returns. Copula models can conveniently separate the leverage effect from the marginal distribution of the return and its volatility. In addition, Copulas provide not only the magnitude of leverage effect but also its underlying structure. The data sets used in this study are intra-day transaction prices for three stock indices: the Standard and Poor's 500 (S&P 500), the Nasdaq Composite Index (COMPQ) and the Russell 2000 Index (RUT). The sample of observations covers the periods from January 02, 1998 to December 30, 2005. The realized volatility is formulated from high frequency data and employed as the daily volatility in this study. We find that large negative returns are followed by large volatility in the next period, implying the presence of leverage effect. Moreover, the leverage effect is found to be changing over time although in a highly persistent manner. Lastly, although both the static and dynamic models can explain the leverage effect in the stock indices, we find that the dynamic models perform better than the static models.

Keywords: Leverage Effect; Copulas; Tail Dependence; Realized Volatility; High Frequency Data.

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1 Introduction

One of the stylized facts of returns of financial assets is the so-called leverage effect (see Black (1976) and Christie (1982)), and, moreover, such correlation tends to change over time. In particular, Christie (1982) studies the negative relation between the ex-post volatility in the rate of returns on equity and the current value of the equity. To this date, the quantitative investigation of leverage effect has been less developed. Instead, the leverage measure has been primarily studied in the context of conditional and stochastic volatility models.

In the context of conditional volatility model, for instance Nelson (1991) proposes an EGARCH model, while Zakoian (1994) proposes a TGARCH model. They report evidence of such negative correlation. The threshold effect is typically interpreted as asymmetric effect, when the threshold is set to zero. A common idea used in such asymmetric models is the leverage effect, in which negative shocks to returns increase the predictable volatility to a greater extent than do positive shocks.

On the other hand, the asymmetric property of the stochastic volatility model is based on the direct correlation between the innovations in both the return and its volatility. In the context of a theoretical continuous-time model, Hull and White (1987) generalized the Black-Scholes option pricing formula to analyze stochastic volatility and the negative correlation between the innovation terms. In empirical literature, extensions of a simple discrete time model due to Taylor (1986) are proposed by Wiggins (1987), Chesney and Scott (1989) and Harvey and Shephard (1996) to accommodate the direct correlation. In the context of univariate models, Jacquier, Polson and Rossi (2004) develop a Bayesian Markov Chain Monte Carlo (MCMC) estimation technique to estimate alternative stochastic volatility model with leverage. Yu (2005) compares the model of Jacquier, Polson and Rossi (2004) with the traditional specification by using the news impact function, and shows that the traditional specification should be used in order to describe leverage.

Thus, in the framework of the conditional volatility models, the leverage effect is typically captured in the conditional volatility equation, while in the context of the stochastic volatility models, it is usually accommodated in the correlation between the innovation driving the return process and the innovation driving the latent volatility process. One of the important drawbacks of the conditional volatility model is that the leverage effect is measured in a linear correlation context. This rules out any nonlinear correlation between

the returns and its latent volatility. Moreover, the correlation is usually measured under the assumptions that the innovation driving the return process is Gaussian in the conditional volatility framework, and the innovations of both the return process and and its latent volatility processes are both Gaussian in the stochastic volatility framework. This assumption apparently is counterfactual, especially for the case of high frequency data for equity returns, which tend to be charaterized by high kurtosis and skewness.

In this paper, we develop copula models to directly measure the interrelationship between the stock returns and its future volatility. Copula models are especially suitable for quantitative measure of leverage effect. First, empirical evidence suggests that the leverage effect tends to occur as a downside effect. This implies that only a large negative return is followed by an increase in its volatility. Traditional measure of correlation can not capture this form of leverage effect. Copula models, on the other hand, can directly measure this effect through measure of tail dependence of the return and its volatility distribution. Second, copula models allow for any types of potential distribution of the returns (fat tails, skewed distribution, etc.) and the volatility. Third, Copula models can conveniently separate the dependence between the return and its volatility through their marginal distributions.

We start the analysis with a mixture of copulas that allows for both upper and lower tail dependence. Our empirical evidence shows that only upper tail dependence of the negative return and its future volatility, in essence, makes up for the leverage effect on equity returns; this leads us to focus our analysis on copulas with only upper tail dependence, such as the Gumbel copula and survival Clayton copula. We find significant leverage effect in the stock indexes: the Standard and Poor's 500 (S&P 500), the Nasdaq Composite Index (COMPQ) and the Russell 2000 Index (RUT).

Moreover, to allow for the time varying dependence between the return and its volatility, we extend our static model to a dynamic one. In the dynamic model, the tail dependence innovates over time following an ARMA type process. We provide evidence to suggest that the leverage effect changes over time but with strong persistency.

The remaining part of this paper is organized as follows: Section 2 describes the methodology used to model the leverage effect in stock returns. Section 3 presents the data and describes how the volatility measure used in this study is calculated. Section 4 presents and discusses the empirical results. Section 5 concludes.

2 The Methodology

Let X_t and Y_t respectively be the negative of the return and its latent volatility. The leverage effect is then defined as the dependence between X_{t-1} and Y_t. If $u = F(x_{t-1})$ and $v = F(y_t)$ are the marginal distribution functions of the return and its latent volatility with the joint distribution function $F(x_{t-1}, y_t)$, then by Sklar's (1959) theorem, there exists a copula $C(.)$, such that for all x, y in R,

$$F(x_{t-1}, y_t) = C(F(x_{t-1}), F(y_t)). \tag{1}$$

Thus, a copula is a joint distribution function of marginal distributions, which are assumed to be uniform on the interval [0,1].

Furthermore, by Sklar's Theorem, a joint distribution can be decomposed into its univariate marginal distributions, and a copula, which captures the dependence structure between the variables X and Y. As a result, copulas allow us to conveniently model the marginal distributions and the dependence structure of a multivariate random variable separately.

Based on the selected copulas, we can define two alternative nonparametric measures of dependence between the two variables, namely the Spearman's ρ and Kendall's τ rank correlation coefficients. Unlike the simple correlation coefficient, these rank correlations do not require a linear relationship between the variables. For this reason, they are commonly studied with copula models. The relationship between Spearman's ρ measure and copulas can be expressed as:

$$\rho = 12 \int_0^1 \int_0^1 uv dC(u, v) - 3 \tag{2}$$

So, the parameter ρ is related to the marginal distributions. We note that there is an exact relationship between the copulas and the Kendall's τ measure, which, for variables X and Y, is defined as the difference between the probability of the concordance and the probability of the discordance. Specifically, the higher the τ value, the stronger the dependence between X and Y. The relationship between Kendall's τ measure and copulas can be stated as:

$$\tau = 4 \int_0^1 \int_0^1 C(u, v) dC(u, v) - 1 \tag{3}$$

From the above expression, we see that the Kendall's τ measure does not depend on the marginal distributions. Therefore, any comparisons between

results obtained using different copula functions should be based on the common Kendall's τ measure, since the ensuing comparison would be made under the same rank dependence.

Another useful dependence measure based on the copulas is a measure of tail dependence of the return distribution, which is used to measure co-movements of variables in extreme situations. Tail dependence measures the probability that both variables are in their lower or upper joint tails. Intuitively, upper (lower) tail dependence refers to the relative amount of mass in the upper (lower) quantile of their joint distribution. Because tail dependence measures are derived from the copula functions, they possess all of the desirable properties of the copulas mentioned above. The lower (left) and upper (right) tail dependence coefficients respectively are defined as:

$$\lambda_l = \lim_{u \longrightarrow 0} Pr[F_Y(y) \leq u | F_X(x) \leq u] = \lim_{u \longrightarrow 0} \frac{C(u,u)}{u}, \tag{4}$$

$$\lambda r = \lim_{u \longrightarrow 1} Pr[F_Y(y) \geq u | F_X(x) \geq u] = \lim_{u \longrightarrow 1} \frac{1 - 2u + C(u,u)}{1-u}, \tag{5}$$

where λ_l and $\lambda r \in [0,1]$. If λ_l or λr are positive, X and Y are said to be left (lower) or right (upper) tail dependent; see Joe (1997) and Nelson (1999).

Different copulas usually represent different dependence structures with the so-called association parameters θ_c which indicates the strength of the dependence. Some commonly used copulas in economics and finance include: the bivariate Gaussian copula, the Student-t copula, the Gumbel copula, the Clayton copula, and their combinations. The Gaussian copula does not have tail dependence, while the Student-t copula has symmetric tail dependence; and the Gumbel copula has only upper tail dependence, while the Clayton copula has lower tail dependence.

To model the leverage effect, we apply various copula models. We started with a mixture of the Clayton and survival Clayton copula. The estimation results indicate a zero weight on the Clayton copula. Thus, in the subsequent analysis, we focus on the copulas with only upper tail dependence. The candidates of such copulas naturally belong to the Gumbel copula and survival Clayton copula. So, we use them in the remaining analysis of the paper.

Gumbel copula:

$$\begin{aligned} C(u,v) &= \exp\{-[(-\ln u)^\theta - (-\ln v)^\theta]^{1/\theta}\}, \\ \text{where } \theta &= 1/\log_2(2 - 2^{\lambda r}). \end{aligned} \tag{6}$$

survival Clayton copula:

$$\begin{aligned} C(u,v) &= u+v-1+[(1-u)^{-\theta}+(1-v)^{-\theta}-1]^{-1/\theta} \\ \text{where } \theta &= -1/\log_2(\lambda r). \end{aligned}$$

In addition, we also allow for time varying in the tail dependence of the return distribution. In particular, following Patton (2006), we propose the following ARMA type process for the innovation of the tail dependence:

$$\lambda r = (1+\exp(-h_t))^{-1} \tag{7}$$

$$h_t = h_0 + \beta h_{t-1} + \gamma \sum_{j=1}^{p} |u_{t-j} - v_{t-j}|. \tag{8}$$

The innovation model contains an autoregressive term designed to capture the persistence in dependence, and a variable which is a mean absolute difference between u and v. It is more positive when the two probability integral transforms are on the opposite side of extremes of the distribution and close to 0 when they are on the same side of extremes. The logistic transformation of the ARMA process is intended to guarantee that the tail dependence parameter lies in the range of the [0.1] interval.

Since the focus of our paper is on the leverage effect, which is the dependence between the return and its volatility, we use nonparametric method for the marginal distributions of the return and its volatility. To avoid any distortion of the parametric assumption of the marginal distributions, we use empirical cumulative distribution function (ecdf) for the margins.

After computing the empirical cumulative distribution functions of the return series, we use a maximum likelihood approach to estimate the copula joint models.

3 Data and Measurement of Realized Volatility

We use daily returns and measure of realized volatility for our models. In particular, we construct a daily realized volatility measure using intra-daily high-frequency returns. The data sets for our empirical studies contain intra-day transaction prices for three stock indices including the Standard and Poor's 500 (S & P 500), the Nasdaq Composite Index (COMPQ) and the Russell 2000 Index (RUT). The sample of observations covers the periods

from January 02, 1998 to December 30, 2005.

Define C_t as the closing price on the trading day t. The daily return r_t is calculated as logarithmic closing price differences in the usual way as:

$$r_t = 100(\log C_t - \log C_{t-1}) \quad t = 1, 2, ..., T$$

Realized volatility, defined as the sum of squared intraday returns over a certain interval (such as a day) has been proposed by Andersen and Bollerslev (1998) and Barndorff-Nielsen and Shephard (2001). This measure provides a consistent estimator of the latent volatility in an ideal market condition. However, in the real market, there are two issues associated with the measurement of daily realized volatility from high frequency return data. One issue arises from the presense of the market microstructure noise in the transaction prices, and the other stems from the presence of nontrading hours.

The market microeconomic structure stems from multiple sources, including discrete trading and bid-ask spread. Due to this noise, the realized volatility measure can be a biased estimator of the latent volatility. As the time interval shrinks toward zero, the variance of the true price process is expected to become independent of any decrease in the market microstructure noise. As a result, the effect of the market structure noise can become more pronounced. This argument suggests that there is a trade-off between the variance and bias of the realized volatility measure. Exploiting this trade-off, Bandi and Russell (2005) obtain a simple formula to produce the optimal time interval of intraday returns which is used to calculate the realized volatility measure. Zhang, Mykland and Ait-Sahalia (2005) also propose a method to correct for the bias of the realized volatility measure by combining two realized volatility calculated from returns with different frequencies.

In this paper, we follow Andersen and Bollerslev (1998) and Bolerslev and Zhou (2006) and simply use the sampling frequency at a 5-minute interval. The reason for this is that a 5-minute frequency is considered as the highest frequency at which the prices are less distorted from the market microstructure noise. Consequently, the 5-minute returns $r_{h,t}$ are constructed as:

$$r_{h,t} = 100(\log P_{h,t} - \log P_{h,t-1}) \quad h = 1, 2, .., H; \quad t = 1, 2, ..., T \tag{9}$$

where $P_{h,t}$ is the price on trading day t sampling at 5-minute level.

The realized volatility can underestimate the latent daily volatility, if we define the volatility on day t as the volatility from the market closing time on day t-1 to that on day t and calculate the realized volatility as the sum of squared intraday returns when the market is open. To discuss this particular issue further, consider a simple price process:

$$dlogP(s) = \sigma(s)dW(s) \tag{10}$$

where $logP(s)$ denotes the log-price of the stock return at time s, and $\sigma^2(s)$ is a spot volatility, which is assumed to have locally square integrable sample paths and independent of the standard Brownian Motion $W(s)$. Then, the volatility on day t is defined as the integral of $\sigma^2(s)$ over an open interval $(t, t+1)$, where a full 24 hours trading is represented by the time interval 1; that is,

$$IV_t = \int_t^{t+1} \sigma^2(s)ds \tag{11}$$

This is known as the integrated volatility.

Although the integrated volatility cannot be observed, we can calculate it using observable high frequency return data. Suppose that we have $H+1$ intraday stock returns during each day, $\{r_{t,h}\}_{h=0}^{H}$, then the realized volatility is defined as the squared sum of returns over day t:

$$RV_t = \sum_{h=1}^{H} r_{t,h}^2 \tag{12}$$

In the ideal market, in the absence of microstructure noise and where the asset were always and continuously traded, the realized volatility measure would be a consistent estimator of the integrated volatility; that is, $RV_t \to IV_t$ as $H \to \infty$. Equivalently, the discretization noise due to $dW(s)$ in the realized volatility vanishes as the time interval goes to zero.

However, in the real market and to the extent that stock markets are open only during part of the trading day, the realized volatility defined above, which measures the open-to-close volatility, can underestimate the integrated volatility when the market is open. To avoid underestimation, we can include the returns on the nontrading hours (overnight and/or lunch time interval), but this can make the realized volatility measure noisy because such returns contain too much discretization noise. Therefore, in this paper, we follow Martens (2002) and construct an alternative volatility measure by scaling

the realized volatility in (12) for the market open period, using some weight, w;[1] that is,

$$RV_t = w \sum_{h=1}^{H} r_{h,t}^2 \tag{13}$$

where $w = 1 + \frac{v_1}{v_2}$, with

$$v_1 = \frac{10000}{T} \sum_{t=1}^{T} (\log P_{H,t} - \log P_{0,t})^2 \qquad v_2 = \frac{10000}{T} \sum_{t=1}^{T} (\log P_{0,t} - \log P_{H,t-1})^2$$

In this paper, we use (13) to construct all of our realized volatilities. Table 1 presents the summary statistics of the daily returns, squared daily returns, constructed realized volatilities and the log of constructed realized volatilities for the three indices: S&P 500, COMPQ and RUT.

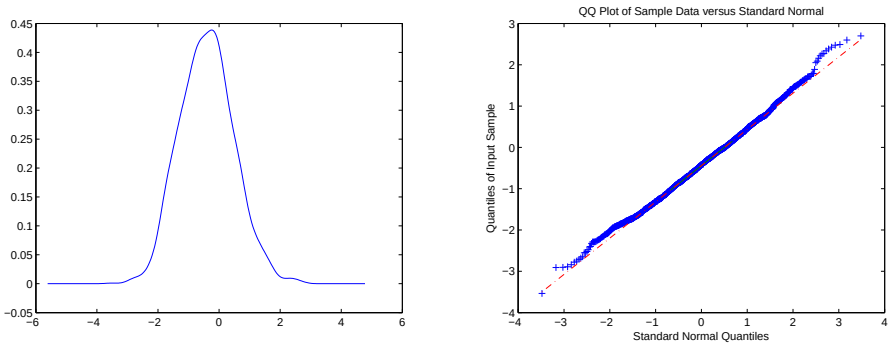
From Table 1, three groups of results stand out in the table. First, the sample mean of realized volatility measure is smaller than that of the squared daily return. This result provides evidence to suggest that there is negative bias in the realized volatility due to nontrading hours. The sample variance of the realized volatilities is also much smaller in magnitude than those of the squared return.

Second, the sample skewness and kurtosis values indicate that the realized volatilities are far from being Gaussian. However, their logarithms are nearly Gaussian. This motivates us to wok with the logarithm of realized volatilities instead of realized volatilities.

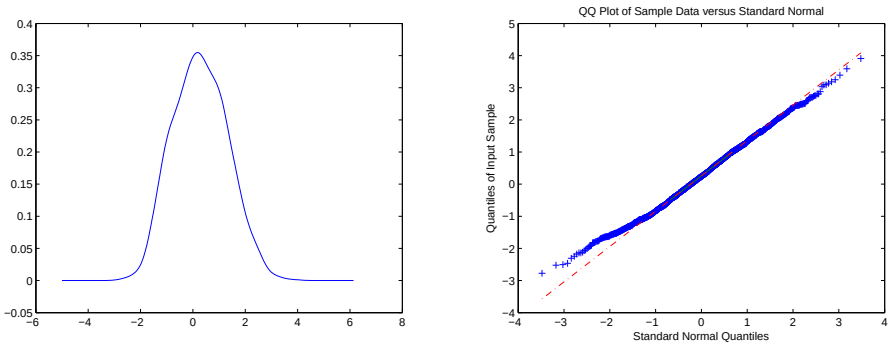
Third, the sample autocorrelation coefficients of the return series up to the fifth lag show that daily return is not autocorrelated, while volatilities, in particular, the log realized volatilities are highly autocorrelated. This result conforms with the phenomenon of volatility clustering known to characterize stock returns. The above findings are in line with those reported in Andersen, Bollerslev, Diebold and Ebens (2001).

[1] An alternative weight was suggested by Hansen and Lunde (2006), where the weight is defined as $w = \sum_{t=1}^{T}(R_t - \bar{R})^2 / \sum_{t=1}^{T} RV_t$, R_t is the daily return, T is the daily sample size, and $\bar{R} = T^{-1} \sum_{t=1}^{T} R_t$. This particular measure ensures that the mean of the scaled realized volatility is equal to the variance of daily returns.

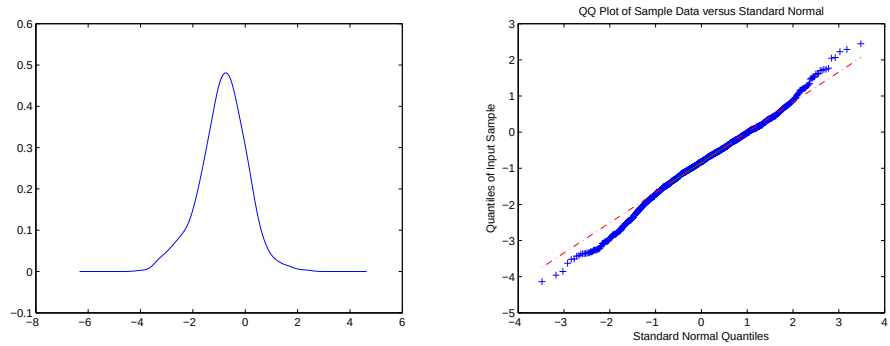
Figure 1: Distribution of log Realized Volatility



S & P 500

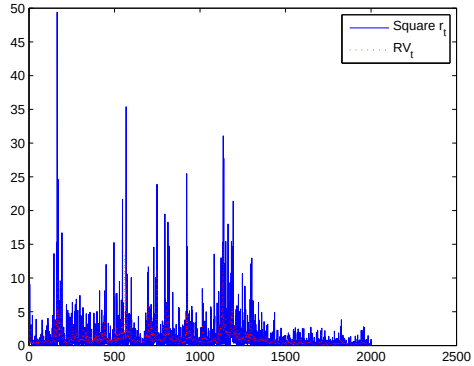


COMPQ

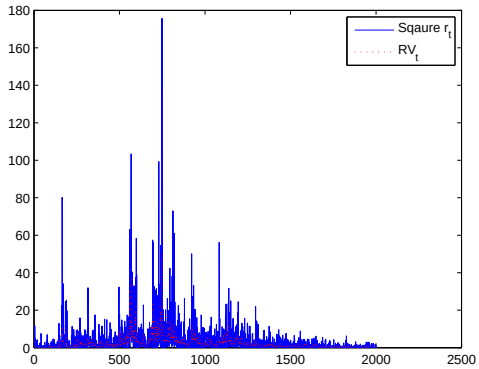


RUT

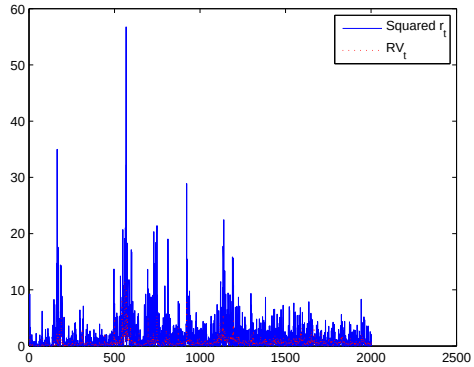
Figure 2: Squared Return Vs Realized Volatility



S & P 500



COMPQ



RUT

Figure 2 plots the squared daily returns and the realized volatilities. We find that both series exhibit high persistency. However, the realized volatilities are much less volatile than the squared daily returns. This result is consistent with the descriptive statistic results reported in Table 1.

4 Dependence Results and Leverage Effect

In this section, we present the results of dependence between the negative of return and its future volatility. As a first step, we examine three conventional measures of dependence: the simple linear correlation coefficient, Spearman's rank correlation and Kendall's rank correlation. As shown in Table 2, the linear correlation coefficient between the negative of the return and its future volatility ranges from 0.21 to 0.26, indicating that a higher loss in the return is associated with an increase of its future volatility. The Spearman's rho measure falls in the range between 0.15 to 0.17, indicating un-neglected rank correlation. Lastly, the Kendall's tau coefficient is around 0.1, suggesting that the probability of concordance is significantly higher than the probability of discordance.

Next, in order to get a sense of the dependence structure in the data, following Knight, Lizieri and Satchel (2005), we construct an empirical copula table. To do this, we first rank the pairs of the negative return series and the realized volatility in the next period in the ascending order and, then, we divide each series evenly into 8 bins. Bin 1 includes the observations with the lowest values and bin 8 includes observations with the highest values. We want to know how the values of one series are associated with the values of the other series. Especially, we want to find out whether high negative returns are associated with high volatility in the next period. Thus, we count the numbers of observations that are in cell (i, j). The dependence information we can obtain from the frequency table is as follows: if the two series are perfectly positively correlated, most observations will lie on the diagonal; if they are independent, then we would expect that the numbers in each cell are about the same; if the series are perfectly negatively correlated, most observations should lie on the diagonal connecting the upper-right corner and the lower-left corner; if there is positive lower tail dependence between the two series, we would expect that more observations in cell (1,1). Lastly, if positive upper tail dependence exists, we would expect to see a large number in cell (8,8).

Tables 3, 4 and 5 show the dependence structure for each of the three indices. For S & P500, the number in Cell (8,8) is 93. This means that out

of 2002 observations, there are 93 occurrences when both the negative S & P500 index return and its volatility in the next period lie in their respective highest 8th percentiles (upper1/8th quantile). This number is the largest among all of the cells, and it is much bigger than the numbers in all of the other cells. We take this as evidence of upper tail dependence, which means that a large negative return is followed by higher volatility. The same phenomenon is observed for COMPQ and RUT. Overall, the tables show strong evidence of leverage effect for all three indices.

Now, we estimate the static upper tail dependence between the negative return and its volatility of the next period, using the Gumbel copula and the survival Clayton copula. The results are shown in Table 6. As seen from the table, there exists significant upper tail dependence between the negative of the return and the future volatility in all three indices. The tail dependence estimated from the Gumbel copula model ranges from 0.18 to 0.21, while that from the survival Clayton copula model ranges from 0.16 to 0.22. This implies that the leverage effect occurs during the extreme downturn periods. Comparing the two copulas, the tail dependence estimates are observed to be similar to the survival Clayton copula model. However, the survival Clayton copula model performs better as it obtains much higher likelihood and AIC values than the Gumbel copula does.

Next, we present the result of the time-varying upper tail dependence, which captures the dynamics of the leverage effect, in Table 7. The first-order autoregressive, AR(1), parameter, β, is positive and statistically significant for all indices in both the Gumbel copula and the survival Clayton copula, indicating strong persistent of the tail dependence. The mean absolute difference variable is negative, but generally not significant (except for the S & P 500 index in the Gumbel copula model). So, it does not seem to affect the innovation of the leverage effect. This result is intuitive. It implies the insignificant occurrences of high returns followed by high volatility or low returns followed be low volatility. Comparing the Gumbel copula with the survival Clayton copula, we observe that they produce very similar results with the estimates of the dynamic parameters very close to each other. Again, the survival Clayton copula gives much higher likelihood and AIC values, indicating a better fit. Lastly, comparing the dynamic model with the static model, we find significant increase in the likelihood and AIC values, suggesting that the dynamic models outperforms the static models.

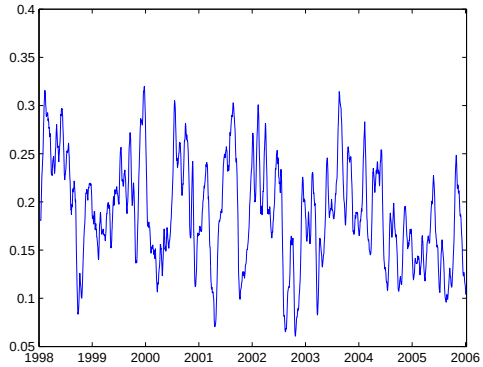
Lastly, we present the dynamics of the leverage effect over time in Figure 3 for the Gumbel copula and Figure 4 for the survival Clayton copula. ¿From

the graph, we see that the mean of the leverage effect is higher than the static case. The leverage effect is both volatile and persistent.

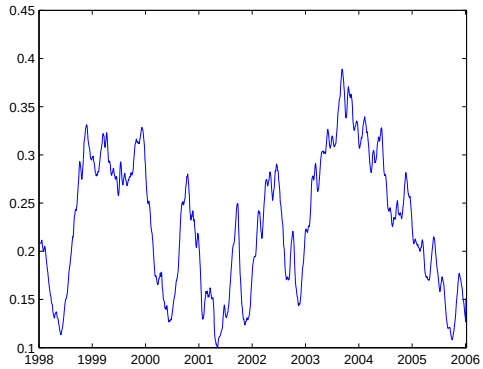
5 Conclusion

In this paper, we have proposed the use of the static and dynamic copulas to model the leverage effect in equity returns. The advantage of using copula models for the analysis of leverage effect is that it can conveniently separate the leverage effect from the marginal distribution of the return and its volatility. Moreover, it provides the information on the underlying structure of the leverage effect. Three US indices were examined in this study. The realized volatility is formulated from high frequency data and employed as the daily volatility in this study. We found that large negative returns are followed by large volatility in the next period, implying the presence of leverage effect. Moreover, the leverage effect was found to be changing over time although in a highly persistent manner. Although both the static and dynamic models can explain the leverage effect in the stock indices, we found that the dynamic models perform better than the static models.

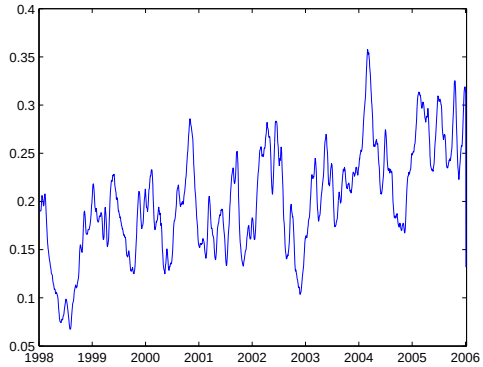
Figure 3: Dynamic Leverage Effect – Gumbel Copula



S & P 500

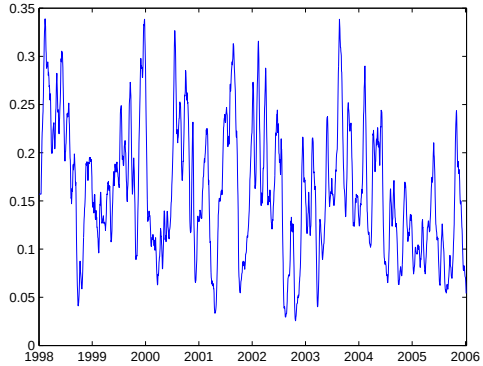


COMPQ

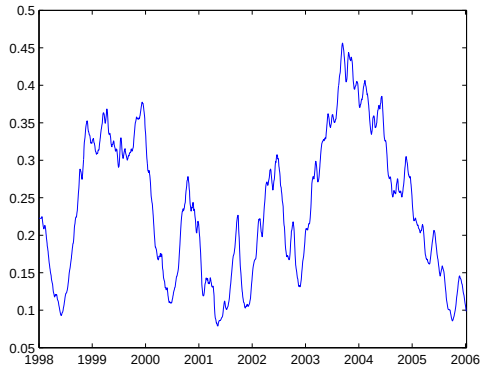


RUT

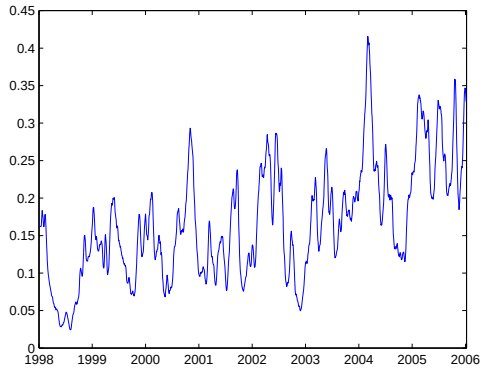
Figure 4: Dynamic Leverage Effect – Survival Clayton Copula



S & P 500



COMPQ



RUT

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Table 1: Summary Statistics

	r_t	r_t^2	RV_t	$\log(RV_t)$
S & P 500				
Mean	0.0118	1.4269	0.9901	-0.4175
Variance	1.4274	8.9395	1.4235	0.7707
Skewness	-0.0013	6.1398	4.7852	0.1786
Kurtosis	5.3892	63.0055	38.2820	3.0928
ACF_1	-0.0206	0.1549	0.6462	0.7689
ACF_2	-0.0228	0.2074	0.5558	0.7381
ACF_3	-0.0303	0.2057	0.4951	0.7003
ACF_4	0.0084	0.1199	0.4807	0.6913
ACF_5	-0.0408	0.1914	0.4617	0.6797
COMPQ				
Mean	0.0135	3.8709	2.2397	0.2640
Variance	3.8727	77.3477	9.2138	1.0649
Skewness	0.0741	7.6827	5.1931	0.1416
Kurtosis	6.1577	102.6833	52.5441	2.6542
ACF_1	-0.0001	0.1963	0.7081	0.8564
ACF_2	-0.0386	0.2737	0.5980	0.8163
ACF_3	0.0004	0.1870	0.5280	0.7874
ACF_4	0.0153	0.1974	0.5205	0.7852
ACF_5	-0.0220	0.1957	0.5142	0.7703
RUT				
Mean	0.0212	1.8009	0.6290	-0.8686
Variance	1.8014	10.4503	0.5823	0.8295
Skewness	-0.1795	5.7030	5.8359	-0.2329
Kurtosis	4.2324	62.7617	57.5792	3.5824
ACF_1	0.0371	0.1352	0.4567	0.6360
ACF_2	-0.0229	0.3369	0.3763	0.6209
ACF_3	0.0350	0.1905	0.3643	0.5840
ACF_4	0.0202	0.2531	0.3352	0.5632
ACF_5	-0.0280	0.1949	0.3336	0.5492

Table 2: Correlation Coefficients

	Leanear corr.	Spearman rank corr.	Kendall rank corr
S & P 500	0.20667	0.15061	0.10176
COMPQ	0.25998	0.15138	0.10186
RUT	0.23261	0.16626	0.11148

Table 3: S & P 500 Joint Frequency Table

S & P500	1	2	3	4	5	6	7	8	Total
1	12	38	49	54	38	29	29	1	250
2	20	36	35	51	48	30	25	5	250
3	23	33	45	43	31	28	43	4	250
4	31	35	29	20	29	43	39	24	250
5	35	30	29	25	32	36	33	31	251
6	49	31	21	19	30	27	32	41	250
7	38	27	26	28	26	33	22	51	251
8	42	20	17	10	16	25	27	93	250
Total	250	250	251	250	250	251	250	250	2002

Table 4: COMPQ Joint Frequency Table

COMPQ	1	2	3	4	5	6	7	8	Total
1	4	30	47	70	44	45	9	0	249
2	16	38	42	49	40	39	21	5	250
3	20	25	45	34	48	40	34	5	251
4	27	36	34	27	43	40	33	10	250
5	29	42	41	23	27	27	41	21	251
6	43	35	16	22	19	26	47	42	250
7	64	33	14	17	18	18	28	59	251
8	47	11	12	8	11	16	37	108	250
Total	250	250	251	250	250	251	250	250	2002

Table 5: RUT Joint Frequency Table

RUT	1	2	3	4	5	6	7	8	Total
1	7	39	54	45	47	33	22	2	249
2	27	45	39	31	27	45	30	6	250
3	33	31	36	41	33	32	29	16	251
4	34	34	29	31	35	27	32	28	250
5	41	28	22	35	35	29	30	31	251
6	38	25	29	25	33	36	34	30	250
7	33	31	29	25	26	26	34	47	251
8	37	17	13	17	14	23	39	90	250
Total	250	250	251	250	250	251	250	250	2002

Table 6: Static Tail Dependence

Data	λ_{Upper}	Likelihood	AIC
Gumbel Copula			
S & P500	0.1809(11.3326)**	74.6433	-147.3866
COMPQ	0.2083(13.2871)**	103.4382	-204.8764
RUT	0.1901 (11.8257)**	80.4917	-158.9834
Survival Clayton			
S & P500	0.1571(6.3917)**	96.3073	-190.6146
COMPQ	0.2226 (8.9632)**	134.3699	-266.7398
RUT	0.1622 (6.3738)**	95.8901	-189.7802

Note: ** indicates significant at the 5% level

Table 7: Dynamic Tail Dependence

Data	h_0	*beta*	*gamma*	Likelihood	AIC
Gumbel Copula					
S & P 500	0.0492	0.8783	-0.7655	80.1524	-154.3048
	(0.6984)	(15.3180)**	(-1.9922)*		
COMPQ	0.0347	0.9749	-0.2217	111.1693	-216.3386
	(1.6114)	(53.8896)**	(-1.6431)		
RUT	0.0298	0.9603	-0.2892	86.5875	-167.1750
	(1.0904)	(31.9851)**	(-1.4399)		
Survival Clayton					
S & P 500	0.1362	0.8583	-1.2729	100.3758	-194.7516
	(0.9968)	(11.3414)**	(-1.7384)		
COMPQ	0.0488	0.9821	-0.2383	140.1000	-274.2000
	(1.0215)	(43.3279)**	(-0.9451)		
RUT	0.0760	0.9572	-0.5033	99.8062	-193.6124
	(0.9623)	(17.5133)**	(-0.9084)		

Note: ** indicates significant at the 5% level